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A STUDY TO INTEGRATE THE HISTORY  
OF MATHEMATICS INTO THE  
GEOMETRY CURRICULUM

by  
Karen M. Laszecki

A Thesis

Submitted in partial fulfillment of the requirements of the  
Master of Arts Degree in the Graduate Division  
of Rowan University  
May, 1997

Approved by \_\_\_\_\_  
John Sooy

Date Approved April 28, 1997

## ABSTRACT

Karen M. Laszecki  
A Study to Integrate the History  
of Mathematics into the  
Geometry Curriculum  
1997  
Dr. John Sooy  
Mathematics Education

The purpose of this study was to integrate the history of mathematics into those topics found in a high school geometry curriculum and to determine the current extent of this integration. During the last decade, the History in Mathematics in Education Conference (1990) and the History and Pedagogy of Mathematics Newsletter (1992) are examples of recent efforts to use the history of mathematics as an integral part of the curriculum.

Fifty-one teachers from forty high schools in southeastern Pennsylvania responded to the History of Mathematics in the Classroom questionnaire, which was developed from a review of literature. Geometry teachers rated the importance of areas in the history of mathematics that may be used in the geometry curriculum, using a scale of 5 (very important) to 1 (not important). The teachers also noted which of the areas they currently include in their curricula. The Spearman rank correlation coefficient of 0.79 shows a moderately high correlation between which areas teachers judged to be important and their practices in the classroom.

Based on the data gathered from the questionnaire results, etymologies, the axiomatic system of Euclid, and various proofs of the Pythagorean theorem are the areas in which the history can be used integrally.

## MINI-ABSTRACT

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The purpose of this study was to integrate the history of mathematics into those topics found in a high school geometry curriculum and to determine the current extent of this integration. Etymologies, the axiomatic system of Euclid, and various proofs of the Pythagorean theorem should be taught as an integral part of a college preparatory geometry course.

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## CHAPTER 1

### Introduction to the Study

#### Introduction

Teachers of mathematics face a continual challenge to develop curriculum that is both interesting and relevant to their students. One way teachers begin to meet this challenge is by staying current with new developments in their field. Another way is by studying the development of mathematical ideas (NCTM, 1969) and their historical impact (NCTM, 1989). Through such a study, the teacher can gain an appreciation of the influence of mathematics on civilization (Fauvel, 1991). The National Council of Teachers of Mathematics (1989) stated that one of its goals for K-12 students is "to learn to value mathematics." Teachers can help students develop an appreciation of the part mathematics has played in the development of our civilization (Fauvel, 1991). In order to accomplish this goal, the teacher must explore how to use history in the mathematics curriculum.

#### Background of the Study

The question of how to use history in the mathematics curriculum appears to have been more widely researched in Great Britain than in the United States. The research conducted in both places will be considered in this study.

According to Swetz (1994), the importance placed on the history of mathematics for teaching was far greater at the turn of the century than it is today. Fauvel (1991)

similarly noted that teaching mathematics from an historical perspective was more widely discussed earlier this century than it is during the 1990's. According to Fauvel, the UK National Curriculum shows no evidence of interest in this topic. This fact has not diminished the efforts of the British Society for the History of Mathematics to incorporate history into mathematics courses; this organization has also cooperated with other groups outside the UK for the sole purpose of using history integrally in mathematics courses.

In 1969, the NCTM published Historical Topics for the Mathematics Classroom, a yearbook which gave some historical background on topics from computation to calculus. It promoted the teaching of mathematics from an historical standpoint as a way to improve curriculum and to increase student interest (NCTM, 1969). The NCTM (1989) later articulated its goal to help students learn to value mathematics as follows:

Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of contemporary society.

If teachers focus on providing students with a variety of learning experiences, then the incorporation of historical topics will find a place in the curriculum.

#### Statement of the Problem

The purpose of this study is to integrate the history of mathematics into those topics found in a high school geometry curriculum and to determine the current extent of this integration.

#### Significance of the Problem

The importance of this study is that it will advance the inclusion of historical topics

in the curriculum beyond biographies and anecdotes. The NCTM (1969) correctly pointed out that biographical materials are readily available to teachers and that students have already demonstrated interest in biographies. While biography will remain essential to the integration, the focus will be primarily on the mathematics.

The NCTM (1989) recommends that students have a variety of learning experiences. Teachers who seek more effective means of teaching mathematics may be more likely to discover the variety of methods for teaching mathematical topics. Using the history of the subject may be one method to explore.

One subject on which there has been much research and discussion is that of gender differences in mathematics. Teachers are challenged to help females reach their mathematical potential; history may be one tool for accomplishing this goal (Fauvel, 1991).

Another area of more recent interest is the multi cultural classroom. Teachers today are more aware of the range of cultural backgrounds and diversity in their classes; they may seek ways to encourage their students to learn more about other cultures. Using the history of mathematics may help students to appreciate the contributions various cultures have made to the development of mathematics (Fauvel, 1991).

The NCTM (1969) maintains that insight into the history can enhance the curriculum and teaching of the subject. Therefore, this study may further contribute to the development of meaningful curriculum. Furthermore, this study should have impact on increasing student interest in the study of mathematics.

### Limitations of the Study

This study is limited to topics found in a high school geometry course. Although all topics in this course have some historical background and significance, this study will make brief references to some topics and other topics will be explored in depth. While revising an entire curriculum from an historical viewpoint is not the focus of this study, it may include recommendations for the inclusion of additional topics.

The resource literature for this study is abundant; however, most of the literature is in the form of texts. Resources from periodicals are minimal, although some articles are cited as parts of a volume of literature.

### Assumptions

Students are expected to have had some exposure to geometric topics, such as basic shapes and area. These topics may have been part of a pre-algebra or algebra I course so that students could learn the various uses for formulas. It is assumed that the students who may benefit from the study are enrolled in an academic geometry course in which proof is discussed and taught. Coordinate geometry is assumed to be an integral part of the course, not a chapter studied in the concluding weeks of the course.

### Definition of Terms

Many of the unusual terms used in this study will be appear as part of a discussion on the importance of etymology. An etymology is defined as "the history of a linguistic form (as a word) shown by analyzing its component parts, and by identifying its cognates

in other languages." Latin or Greek roots of mathematical terms, therefore, may be unfamiliar.

### Procedures

The first task is to review a current geometry textbook for an overview of the topics that are common to a geometry curriculum. The next process, which is the main goal of this study, is to research and discuss how to develop these topics for instruction, using the history of the topic as the framework. This discussion may sometimes appear out of order with a typical table of contents; however, it is possible to teach some topics out of sequence. Finally, geometry teachers from selected schools in Bucks, Chester Delaware, and Montgomery counties will have the opportunity to respond to a questionnaire. The questionnaire will be designed to answer the question "what is the current extent of the integration of historical topics into a college preparatory geometry curriculum?"

This study will attempt to clarify, for the geometry teacher, which historical topics can be used, and where and when to use these topics. Another question of interest to this study is what teachers are doing currently to integrate the study of the history of mathematics into the curriculum. Their beliefs, opinions, and experiences will provide additional insight into the problem.

## CHAPTER 2

### Review of Literature and Research

#### Introduction

There is no shortage of literature on the topic of the history of mathematics. Chronological texts and biographies are accessible to the teacher and are a natural starting point for the incorporation of history into the mathematics curriculum. The teacher has the task of sorting through these detailed accounts to find those aspects of the history that are relevant to the classroom experience. This process, while both interesting and enlightening, can be quite exhausting; the teacher may seek something more concrete. The NCTM's Historical Topics for the Mathematics Classroom may prove to be a more efficient means of selecting relevant topics. The teacher can quickly locate those topics which have historical background and are part of the curriculum already in place. The NCTM (1969) chose the mathematics as the focus of this work and left the decision of how to bring history into the classroom in the capable hands of the teacher.

Rickey (1992) lists some of the ways the teacher can use the history of mathematics in the classroom:

- To introduce a new topic
- History of specific concepts
- History of notation
- Etymology of terms

Pictures of mathematicians  
Biography: Identify every individual named  
Quotations by famous mathematicians  
Anecdotes  
Title pages from textbooks  
Problems from old textbooks  
As a way to discuss advanced and modern topics

Swetz (1992) suggests that "this can, and should, be done unobtrusively" and " as an integral part of the lesson."

### Review of Literature

Upon reviewing a standard high school geometry curriculum and the literature related to the history of mathematics, there appear to be many parts of the curriculum which can be enhanced by including historical background. Notation, etymology, the concept of measurement, and mathematicians (Rickey, 1992) are the main categories for discussion.

#### Notation

Notation is one form of written communication which high school geometry students are expected to use. Mathematical notation consists of characters, symbols, and abbreviations (Merriam-Webster, 1994). For students who further their study of mathematics, notation increases in quantity and in complexity.

In Cajori's A History of Mathematical Notations (1928), the goal is "to give not only the first appearance of a symbol and its origin, but also to indicate the competition encountered and the spread of the symbol among writers in different countries." By studying symbols, students will learn that the geometric symbols they use were not used by Euclid and that most of these symbols were developed more than one-thousand years

after Euclid wrote Elements (Cajori, 1928). To understand the effect of symbols on communication, it may be useful to examine how two mathematical expressions can look completely different symbolically, but express exactly the same idea.

Students are expected to use various pictographs, which are pictures that represent geometric concepts. Some examples are  $\triangle$  for triangle,  $\bigcirc$  for circle,  $\parallel$  for parallel lines,  $\frown$  for arc, and  $\perp$  for right angle. Another type of symbol is the ideogram, which is symbol that represents a thing or idea such as  $\cong$  for congruence and  $\sim$  for similar (Cajori, 1928).

Greek letters have two important uses in a high school geometry course. First, they may be used to name angles, especially during that part of the course in which the students have their first exposure to right triangle trigonometry. In 1923, the National Committee on Mathematical Requirements recommended the use of small Greek letters to represent angles. This committee also recommended the symbol  $\angle$  for angle; this symbol first appeared as  $<$  in 1634 and was later modified, perhaps because it had been used earlier to mean "less than" (NCTM, 1969). The most important Greek letter for high school geometry students is  $\pi$  (pi). Students will discover this ratio as they develop the formula for the circumference of a circle. Beckman (1971), in A History of  $\pi$ , notes that the first appearance of  $\pi$  to represent the ratio of the circumference of a circle to its diameter occurs in a 1706 work by William Jones. The symbol may have been an abbreviation for the words periphery or perimeter. Only after Euler used the symbol, in a work published thirty years later, did other writers and mathematicians adopt it.

### Etymology

Etymologies can be used periodically to introduce new words with some historical



reference. Simply stated, etymology is the study of word origins. In high school geometry, students will learn and use numerous new terms and their definitions. The etymology can often show a connection between the word and the English definition. Some examples are:

chord: chorda (string)  
circumscribe: circum- (around), scribere (to write)  
congruent: congruere (to come together, agree)  
diagonal: dia- (through, apart), gonia (angle)  
geometry: geo- (earth), metron (measure)  
hypotenuse: hypo- (under), teinein (to stretch)  
inscribe: in- (in), scribere (to write)  
isosceles: isos- (equal); skelos (legs)  
parallel: para- (beside), allos (one another)  
polygon: poly- (many), gonon (angle)  
scalene: skolios (crooked), skelos (leg)  
sine: sinus (curve)  
symmetry: syn- (with, together with), metron (measure)  
tangent: tangere (to touch)  
trapezoid: tra- (four), peza- (foot)

The above prefixes and roots are of Latin or Greek origin (Merriam, 1994).

Though most of the students will not have studied Greek or Latin, they may be able to connect the roots with other English words they know. The students can easily research the above word origins independently. The teacher can also introduce these new terms, their origins, and their definitions as a natural part of a lesson.

### Measurement

In Surveying Instruments: Their History and Classroom Use, Kiely (1947) states that "the need for measurement must have been felt from the very beginning of human existence." What followed was a detailed description of the uses of measuring instruments in various cultures and their histories. Kiely found that in the earlier part of this century,

several committees promoted the use of surveying instruments by high school students in an outdoor setting. In "The Place of Mathematics in Secondary Education", the NCTM and the Mathematical Association of America listed some essentials in the field of geometry. One was that students were expected to demonstrate proficiency using the technique of indirect measurement. This joint commission promoted its cause as follows:

"Instruments for surveying, especially a transit, have considerable usefulness in teaching parts of geometry and trigonometry. The interest of many pupils is increased by means of practical problems and an instrument makes it possible for pupils to secure their own data for variety of exercises" (Kiely, 1947).

In Geometry: An Integrated Approach, there are projects in which students can make two modern surveying instruments, the hypsometer and the clinometer, in a lesson on indirect measurement (Larson, Boswell, and Stiff, 1995). But Kiely (1947) recommends that students use older instruments for three reasons: (1) they were more simply constructed, (2) they are useful for the study of astronomy, and (3) they show a closer connection to the history of measurement. In either case, the origins of these instruments and the problems for which they were designed unveil opportunities to bring history into the mathematics classroom.

#### Mathematicians

The NCTM (1969) stated that "...students are interested in the 'men of mathematics' as people. Anecdotes and incidents in the lives of these men are often of great value in the classroom." Many secondary mathematics textbooks contain biographical inserts; these often have little obvious connection to the current topic. There are mathematicians who have had a direct impact on the development and study of

geometry. In a high school geometry course, these men are Euclid, Archimedes, Descartes, Pythagoras, Thales, Eratosthenes, and Euler. Their biographies can be used in conjunction with the study of their contributions to the field of geometry.

Euclid. Motz and Weaver (1993) state that Euclid's Elements is "perhaps the most famous and successful textbook of all time." It is a compilation of the works of earlier mathematicians and the work of Euclid. He created a structure for geometry which consists of definitions, axioms, postulates, and theorems. It is important to note that, until Elements, geometry existed as a practical tool, especially in Egypt. The Greeks were primarily responsible for making mathematics an abstract subject. Euclid also used the *reductio ad absurdum* proof, otherwise known as indirect proof or proof by contradiction, which shows that the assumption that the theorem to be proved is false leads to a contradiction.

Archimedes. Students may know about Archimedes from their study of the hydrostatic principle in their physical science class; many may also be able to relate the legend surrounding "Eureka!" His impact on history of mathematics may not be well known. Archimedes designed the catapults and other mechanical devices which defended the city of Syracuse from Roman attacks for nearly two years (Motz and Weaver, 1993). His method of calculating  $\pi$  is the earliest known and can be summarized as follows:

"The perimeter of a regular polygon of  $n$  sides inscribed in a circle is smaller than the circumference of a circle, whereas the perimeter of a similar polygon circumscribed about the circle is greater than its circumference. By making  $n$  sufficiently large, the two perimeters will approach the circumference arbitrarily closely, one from above, the other from below (Burton, 1991)."

This process is sometimes referred to as the "method of exhaustion." Students can explore this method as well as the meaning of its name.

Déscartes. In many high school geometry textbooks, the study of coordinate geometry often appears among the final chapters. More recently, textbook writers have begun to integrate coordinate geometry with other topics throughout the book. Students may first learn of Déscartes in their study of the coordinate, or Cartesian, plane. It was Déscartes who designed the coordinate system as a way to locate points on a plane; this system helped to illustrate the concept of a function and to prove the properties and relationship of geometric figures in a plane (Motz and Weaver, 1993).

Pythagoras. Although he is more famous for the theorem which bears his name, Pythagoras was the first of the Greek mathematicians to use deductive reasoning and proof; this would influence Euclid in the writing of Elements (Motz and Weaver, 1993). The study of the Pythagorean theorem is perhaps the most significant topic in a high school geometry course. Many students are familiar with this theorem by the time they enter geometry. In a traditional geometry course, the proof which uses similar triangles is normally studied. But there are numerous proofs of this theorem, some of which pre-date Pythagoras, such as the Babylonian and Chinese proofs (Burton, 1991). Former U.S. President Garfield was also credited with a proof which can be studied once the students have derived the formula for the area of a trapezoid. Many of these proofs are well within the range of abilities of a high school student. Another contribution of Pythagoras is his formula for the harmonic mean, which shows that "three equally taut strings on stringed instruments will sound harmonious if the length of the middle string is equal to the

harmonic mean of the lengths of the shortest and longest string (Larson, et al., 1995)."

Students who are musically inclined may be interested in applying this concept.

Thales. The main accomplishment attributed to Thales was his calculation of the height of the Great Pyramid by the method of indirect measurement (Burton, 1991).

Students will be able to work this problem just as Thales did. Thales was also credited for solving the problem of measuring the distance from a ship at sea to the shore. It is believed he used the properties of congruent and similar triangles to solve this problem (Burton, 1991).

Eratosthenes. The most outstanding contribution of Eratosthenes to geometry is his method for finding the circumference of the earth (Burton, 1991). Students can use their understanding of parallel lines and angles to follow this problem. They can compare Eratosthenes' result with the true value.

Euler. Though the credit for bringing the symbol  $\pi$  into mathematical notation goes to Euler, the Euler line should be of special interest in high school geometry. Students can find this line by constructing the medians, altitudes, and perpendicular bisectors of a triangle. The three points of intersection of these lines lie in the Euler line (Larson, et al., 1995).

The biographies of these mathematicians can include pictures, anecdotes, and quotations, as suggested by Rickey (1992). These are a natural component of the lesson in which the mathematics is the focus.

#### Related Research

Perhaps the most notable research on integrating the history of mathematics into

high school mathematics topics began at the History in Mathematics Education conference in 1990. This conference took place at Leicester University, under the direction of The British Society for the History of Mathematics and the London Mathematical Society. Fauvel spoke of the need to have reports on the success of this integration in the classroom. According to Bibby (1990), teachers from France, Greece, the Netherlands, and Israel reported that work in this area is already under way. One point of emphasis at the conference was "the difference between using history *integrally* and dealing with the history of mathematics as an *adjunct* to an otherwise history-free mathematics course."

The most tangible evidence of research in this area was published in France; the IREM (Institutes for Research on the Training of Mathematics) Papers contain "accounts of teachers' experiences in using historical source materials in the classroom" and "reproducible material for teachers to try (Bibby, 1990)." McLean (1990) recommends this resource as valuable for any mathematics department.

Other evidence that the subject of the history of mathematics in the classroom is alive is the HPM (History and Pedagogy of Mathematics) Newsletter, which Rickey (1992) has said is quite popular. Rickey envisions the HPM as an ever-growing resource for the classroom teacher who needs materials specific to using the history in the classroom.

One use of the history of mathematics for students took place when Dr. Marie Francoise Jozeau (1989) directed her students in the recreation of the measuring of the arc of meridian. The Paris Academy of Sciences did this at the end of the eighteenth century

"to determine the unit of a new measurement system -- the metric system (Jozeau and Grégoire, 1992)." Using their knowledge of geometry and trigonometry combined with the historical background of the problem, the students participated in this project to "measure the earth and universe". They later demonstrated their findings to the local public.

In 1923, The Mathematical Association reported that for grades ten, eleven, and twelve "historical and biographical material should be used throughout to make the work more interesting and significant" (Kiely, 1947). In 1940, the Mathematical Association of America and the NCTM cautioned "the history of mathematics should not supersede mathematics; it should supplement its study" (Kiely, 1947). Current attempts to integrate the study of the history of mathematics into the classroom is evidence that, even toward the end of the twentieth century, educators are trying to answer the call of their predecessors.

## CHAPTER 3

### Procedures

#### Introduction

One goal of this study is to integrate the history of mathematics into those topics found in a high school geometry curriculum. A review of related literature and research in Chapter 2 provides some foundation for exploring this question. The other goal is to learn to what extent geometry teachers currently integrate history into their curricula. A survey of teachers was developed to answer this question. The following is a review of the procedures used in the discussion of the research and in the development of the survey.

#### Literature and Research

A review of the various texts related to the history of mathematics was the motivation for this study. Geometry was chosen above other mathematical subjects so that the results of this study could be put to use either in the classroom or in further research. The next phase of this study began with a review of the student textbook Geometry: An Integrated Approach. This text, combined with the history resources, was a tool for the selection of topics with some historical background. After the selection of topics was complete, the author sought writings specific to the problem of this study. Rickey (1992) contributed to the structure of the discussion in Chapter 2. Bibby (1990)



and Fauvel (1991) provided evidence that other teachers have begun to research the problem of integrating history into the mathematics curriculum.

### The Development of the Survey

Sample Population. The sample population in this study is limited to teachers of college preparatory geometry in southeastern Pennsylvania. Forty high schools from Bucks, Chester, Delaware, and Montgomery counties were selected at random and contacted for this study.

Survey. The survey was divided into three parts which were headed Opinion, Practice, and Comments. In Part I, the outcome of the research in Chapter 2 was transformed into a list of areas of study. Teachers judge the importance of these areas on a scale of 5 to 1, 5 being very important and 1 being not important. In the second part of the questionnaire, the teachers would indicate which of these areas of study are a part of the Geometry course they currently teach. A final section for the teachers' comments was also included.

A committee of five mathematics teachers read the questionnaire and offered their comments and suggestions for revision. The main focus of the ensuing discussion was the validity of the survey. One area previously included by the author was judged as not valid and was later deleted. Examples under the category of Etymologies were added to provide further clarification. The committee agreed that the survey would be easy to read and take a short time to complete. A copy of the survey is located in Appendix A.

### Distribution of the Survey

The surveys were mailed to the supervisor of the mathematics department of each high school selected on March 5, 1997. In each mailing there was a cover letter addressed

to the mathematics supervisor, a cover letter to each geometry teacher, a copy of the survey for each teacher, and addressed, stamped return envelopes for each response. Each supervisor received three surveys so that he/she could distribute to more than one teacher, if possible. Copies of the cover letters are found in Appendix A.

The cover letter to the mathematics supervisor included the statement of the problem and a request to distribute the survey materials to the geometry teacher(s). The letter to each teacher related the problem of the study and its current importance. The teachers were asked to respond by March 17, 1997. The questionnaire was confidential; therefore, the survey results will be mailed to the supervisor when available.

## CHAPTER 4

### Analysis of Data

#### Introduction

The purpose of this study is to integrate the history of mathematics into those topics found in a high school geometry curriculum and to determine the current extent of this integration. The latter part of this problem will be the focus of this chapter. The author will report on and analyze the data gathered from the returned questionnaires, which were described in Chapter 3.

#### Analysis of Data

The data collected will be discussed according to the three parts into which the questionnaire was divided: opinion, practice, and comments. Fifty-one questionnaires were returned from the forty schools contacted in Bucks, Chester, Delaware, and Montgomery counties in southeastern Pennsylvania. The names of these schools are listed in Appendix B.

Opinion. In this section of the questionnaire (see Appendix A), teachers rated the importance of areas in which "the history of mathematics may be an integral part of a lesson" according to the following scale:

5 - Very Important

4 - Important

3 - Somewhat Important

2 - Of Little Importance

1 - Not Important

The response totals for each of these areas are located in Appendix C of this study.

In order to determine the ranking of each area in terms of its importance, the weighted mean was calculated ( $\Sigma (f \cdot x) / \Sigma f$ ) as follows. Each score (x) was multiplied by the frequency (f) of that response. The resulting products were summed; this sum was then divided by the total frequency of the weights ( $\Sigma f$ ). The resulting number is the weighted mean. The ordered rankings listed in Table 1 reflect the importance of each area as determined by its weighted mean.

**TABLE 1**

**Ranking Of Areas In Which The History of Mathematics May Be An Integral Part  
Of A Lesson As Determined By High School Geometry Teachers**

Area	Mean Score
Connection between word parts and the English definition	3.70
Latin or Greek prefixes and roots	3.70
Euclid's <u>Elements</u> as the foundation for Geometry as an axiomatic system	3.58
Various proofs of the Pythagorean Theorem	3.57
Descartes' development of a coordinate system	3.51
Indirect Measurement	3.39
Anecdotes	3.29
Eratosthenes' method for finding the circumference of the Earth	3.10

TABLE 1, continued

Area	Rank
Thales' calculation of the height of the Great Pyramid	3.02
Origin of geometric symbols and notation	3.02
Biographies	3.00
Archimedes' calculation of $\pi$	3.00
Uses for the Greek alphabet	2.92
Surveying instruments	2.86
Pythagoras' formula for the harmonic mean	2.85
Pictures of mathematicians	2.82
Euler Line	2.72
Euclid's use of the reductio ad absurdum proof	2.58

According to the above rankings, none of the areas was determined to be very important or important to a high school geometry curriculum. The areas lie in a range between "Of Little Importance" to "Important" in a high school geometry curriculum. The practices of these teachers and their comments should provide further clarification and explain the above results.

Practice. The data gathered from this part of the questionnaire will show which of the areas are part of the geometry curricula of these teachers. Table 2 lists the areas in the history of mathematics and the percentage of respondents who cover these areas in the geometry course which they currently teach.

TABLE 2

**Areas In The History Of Mathematics Which Are Part Of A College Preparatory  
Geometry Curriculum (including percentage)**

Area	Number of Responses	Number of Respondents	%
Connections between word parts and their English definitions	38	50	76
Latin or Greek prefixes/roots	37	50	74
Various proofs of the Pythagorean Theorem	36	49	73
Euclid's Elements as the foundation for geometry as an axiomatic system	31	50	62
Anecdotes	30	49	61
Indirect Measurement	26	49	53
Descartes' development of a coordinate system	26	49	53
Origin of geometric symbols and notation	20	50	40
Biographies	20	50	40
Uses for the Greek alphabet	19	50	38
Pictures of Mathematicians	17	50	34
Archimedes' calculation of $\pi$	15	49	31
Euler Line	14	47	30
Euclid's use of the reductio ad absurdum proof	12	48	25
Thales' calculation of the height of the Great Pyramid	11	49	22
Eratosthenes' method for finding the circumference of the Earth	11	49	22
Pythagorean formula for the harmonic mean	10	48	21
Surveying instruments	8	50	16

The above percentages reveal that geometry teachers do integrate the history of mathematics into their curriculum. They also show that a small percentage of these teachers cover indirect proof or discuss the application of geometry to solving the historical problems of Archimedes, Euler, Thales, and Eratosthenes. A higher percentage of teachers cover etymologies, proofs of the Pythagorean Theorem, and the axiomatic structure of Euclid. One explanation for this fact is that, history aside, these topics are essential elements of a college preparatory geometry curriculum.

Other Findings Upon examining the Tables 1 and 2 together, there appears to be some correlation between the areas teachers judge to be important and those that they include in their curricula. Table 3 shows the areas in the order presented on the questionnaire, their rankings as shown in Tables 1 and 2, and the difference between these rankings.

**TABLE 3**

**Rankings For Areas In The History Of Mathematics Important To And Part Of A College Preparatory Geometry Curriculum**

Area	Rank In Table 1	Rank In Table 2	Difference in Ranks
Origin of Geometric Symbols and Notation	10	8	2
Uses for the Greek alphabet	13	10	3
Connections between word parts and English definition	1	1	0
Latin/Greek Prefixes/Roots	2	2	0
Indirect Measurement	6	6	0
Surveying Instruments	14	18	-4
Pictures of Mathematicians	16	11	5
Biographies	11	9	2

TABLE 3, continued

Area	Rank In Table 1	Rank In Table 2	Difference in Ranks
Anecdotes	7	5	2
Euclid's <u>Elements</u> as the foundation for Geometry as an axiomatic system	3	4	-1
Euclid's use of the reductio ad absurdum proof	18	14	4
Archimedes' calculation of $\pi$	12	12	0
Déscartes' development of a coordinate system	5	7	-2
Pythagoras' formula for the harmonic mean	15	17	-2
Thales' calculation for the height of the Great Pyramid	9	15	-6
Eratosthenes' method for finding the circumference of the Earth	8	16	-8
The Euler Line	17	13	4
Various proofs of the Pythagorean Theorem	4	3	1

To measure the correlation between the areas teachers think are important and those areas which they include in their curricula, the Spearman rank correlation coefficient (Triola, 1992) was calculated using the formula:

$$r_s = 1 - [6 \sum d^2] / [n (n^2 - 1)],$$

where  $d$  is the difference between the ranks, and  $n$  is the number of pairs of data. For the eighteen pairs of data in Table 3, the rank correlation coefficient  $r_s$  was 0.79. This result



shows a moderately high positive correlation between the importance of areas in the history of mathematics and the coverage these areas receive in the classroom.

Comments. Out of 51 teachers, 25 included comments. They offered opinions, questions, and ideas in response to the author's request to include ways they use the history of mathematics and/or envision its use. Some of the teachers described assignments in which their students were asked to do the following:

- create bulletin boards on which the accomplishments of famous mathematicians are described
- write research projects on the lives of great mathematicians
- create posters (to be displayed weekly) showing contributions of Greek geometers
- present oral group reports on famous mathematicians' accomplishments
- use surveying instruments (transits, tapes) to solve problems
- use technology to access historical information on mathematics
- read NCTM publications
- study Egyptian, Mayan, and Chinese contributions to geometry
- apply the Königsberg bridge problem to a real-life problem

Two teachers also included a copy of the research project information they distribute to their students. One teacher commented that many of the topics in the project are pertinent to the survey completed. Upon reading both project sheets, it is clear that students would encounter all of the areas in which mathematicians are the focus. Some teachers noted that projects like the ones described above are required for the honors level, extra credit, or enrichment, if time allows.

One factor which some teachers cited is time constraints which many teachers of mathematics face. These constraints were due to the quantity of material in geometry, the pressure to complete it, and the importance of preparing their students for district-wide final examinations.

## CHAPTER 5

### Summary, Conclusions, and Recommendations

#### Introduction

The purpose of this study is to integrate the history of mathematics into those topics found in a high school geometry curriculum and to determine the current extent of this integration. The review of the literature and the research was essential to developing a foundation for the discussion of this problem. The topics that emerged from the research were the basis for the questionnaire, which was needed to address the second part of the problem. The author tried to answer this question through: (1) an examination of opinions on the importance of these topics in a high school geometry curriculum; (2) a survey of which topics teachers currently include in the curriculum; and (3) some input, in the form of comments, from the teachers who completed the questionnaire.

#### Summary

The analysis of the data gathered and displayed in the three tables shows that the following topics from the history of mathematics should be included in a college preparatory geometry course:

- Connections between word parts and English definitions
- Latin or Greek prefixes and roots
- Euclid's Elements as the foundation of Geometry as an axiomatic system

- Various proofs of the Pythagorean theorem

Other areas in the history of mathematics which may be included are anecdotes, indirect measurement, and Descartes' development of a coordinate system; for more than fifty percent of the teachers surveyed include these areas in their curricula.

Fifty-one teachers from forty school districts in southeastern Pennsylvania responded to the questionnaire. After a review of the opinion section of each questionnaire, a weighted mean was calculated for each of the eighteen areas to rank them in order of importance. Twelve of the eighteen areas ranked between 3.00 (somewhat important) and 4.00 (important). The five areas that ranked between 3.50 and 4.00 (etymologies, axiomatic system, Pythagorean Theorem, and coordinate geometry) are closely related to those topics which are part of a high school geometry curriculum (definitions, proof, right triangles, and connections to algebra).

The central part of the questionnaire helped to decide which areas are part of the curriculum taught now. Again, etymologies, the Pythagorean theorem, and the axiomatic structure of Euclid's geometry were taught by the highest percentage of teachers.

Teachers who commented gave descriptions of the types of activities in which their students engage that have history at its core. They require many of these activities at the honors level; at other levels these are completed for extra credit or for enrichment. These include bulletin boards, posters, research reports, and oral reports. A key concern expressed by teachers is the pressure of a packed curriculum and the time it takes to implement, leaving little or no space for enrichment.

### Conclusions

The analysis of the data collected from the questionnaire showed the following:

1. Teachers have an appreciation for the history of mathematics enough to include it as required part of their curriculum and, if not, at least as enrichment.
2. Teachers believe that the areas of the history of mathematics that are linked to the overall curriculum are more important than the more isolated areas.
3. Teachers are more likely to examine the history of mathematics as it relates to topics that are at the core of the high school geometry curriculum than to study specific historical problems.
4. Teachers are struggling to complete an abundant, intense curriculum leaving little time to consider the integration of historical topics, even if they do serve to enrich their curriculum.
5. Most of the activities described by the teachers are more additional to the geometry curriculum than integral.

### Recommendations

The author makes the following recommendations as a result of both the review of the literature and research and the analysis of the data collected from the survey.

1. Teachers who are interested in making the history of mathematics an integral part of their curricula examine the IREM papers to learn what their colleagues abroad are doing.
2. Research similar to that of this study should be conducted on a wider scale, including more topics and a larger group of teachers.
3. Teachers consider whether some of the more specific areas from the history of mathematics, such as those by Pythagoras, Thales, Archimedes, Euler, and Eratosthenes can be an integral part of their curricula.

4. Teachers who are interested in the problem continue to communicate and share ways they effectively integrate history into the geometry curriculum.
5. Geometry students who have completed the recommended activities be surveyed to learn how they may have benefited from such work.

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**APPENDIX A**  
**COVER LETTERS**  
**AND**  
**HISTORY OF MATHEMATICS IN THE CLASSROOM QUESTIONNAIRE**

Cheltenham High School  
Rices Mill Road and Carlton Avenue  
Wyncote, PA 19095  
March 5, 1997

Mathematics Supervisor  
Name of High School  
Address of High School  
City Name, PA 00000

Dear Mathematics Supervisor:

I am writing to ask your assistance in contacting teachers who currently teach geometry (college preparatory). I am researching ways to integrate the history of mathematics into a high school geometry curriculum. The input of my colleagues is an important part of my study. At your earliest convenience, would you please forward the enclosed packet to the geometry teacher(s) of your choosing in your department?

Your support and cooperation will be greatly appreciated. If you would like to contact me, please call Cheltenham High School (215-881-6400). Thank you for your time and effort.

Sincerely,

Karen M. Laszecki

Cheltenham High School  
500 Rices Mill Road  
Wyncote, PA 19095  
March 5, 1997

Dear Geometry Teacher:

I am a geometry teacher and am currently researching ways to integrate the history of mathematics into the curriculum. I would appreciate your help in researching this idea.

The question of how to integrate the history of mathematics into a high school mathematics curriculum appears to be of particular interest to high school teachers in Europe. During the past five years, committees from various countries have worked to find ways to make the history an integral part of their lessons. Some national curricula require that the history of the subject be studied in high school.

I would value your opinion on the subject and your input in determining to what extent this integration already takes place in the classroom. Please complete the attached questionnaire and return it to me by March 17, 1997. I have provided an addressed, stamped envelope for your convenience.

Since this survey is confidential, I shall send the results to your supervisor when my research is complete. Thank you very much for your support.

Sincerely,

Karen M. Laszecki

## **Questionnaire: History of Mathematics in the Classroom**

### **PART I: OPINION**

**Directions:** The following is a list of areas in which the history of mathematics may be an integral part of a lesson. Please respond to the importance of each area of study by circling the appropriate number according to the scale below.

#### **Scale**

- 5 - Very Important
- 4 - Important
- 3 - Somewhat Important
- 2 - Of Little Importance
- 1 - Not Important

#### **NOTATION**

- |  |   |   |   |   |   |
|--|---|---|---|---|---|
| 1. THE ORIGIN OF GEOMETRIC SYMBOLS<br>AND NOTATION | 5 | 4 | 3 | 2 | 1 |
| 2. USES FOR THE GREEK ALPHABET                     | 5 | 4 | 3 | 2 | 1 |

#### **ETYMOLOGIES (WORD ORIGINS)**

- |  |   |   |   |   |   |
|--|---|---|---|---|---|
| 1. CONNECTIONS BETWEEN WORD PARTS<br>AND THE ENGLISH DEFINITION<br>(e.g. isos- (equal); skelos (legs) for isosceles)   | 5 | 4 | 3 | 2 | 1 |
| 2. LATIN OR GREEK PREFIXES AND ROOTS<br>(e.g. circum- (around), scribe (to write) are Latin<br>roots for circumscribe) | 5 | 4 | 3 | 2 | 1 |

#### **MEASUREMENT**

- |                          |   |   |   |   |   |
|--------------------------|---|---|---|---|---|
| 1. INDIRECT MEASUREMENT  | 5 | 4 | 3 | 2 | 1 |
| 2. SURVEYING INSTRUMENTS | 5 | 4 | 3 | 2 | 1 |

#### **MATHEMATICIANS**

- |  |   |   |   |   |   |
|--|---|---|---|---|---|
| 1. PICTURES OF MATHEMATICIANS  | 5 | 4 | 3 | 2 | 1 |
| 2. BIOGRAPHIES   | 5 | 4 | 3 | 2 | 1 |
| 3. ANECDOTES   | 5 | 4 | 3 | 2 | 1 |
| 4. EUCLID'S <u>ELEMENTS</u> AS THE FOUNDATION<br>FOR GEOMETRY AS AN AXIOMATIC SYSTEM | 5 | 4 | 3 | 2 | 1 |

Next Page ----->

5. EUCLID'S USE OF THE REDUCTIO AD ABSURDUM PROOF	5	4	3	2	1
6. ARCHIMEDES' CALCULATION OF $\pi$	5	4	3	2	1
7. DÉSCARTES' DEVELOPMENT OF A COORDINATE SYSTEM	5	4	3	2	1
8. PYTHAGORAS' FORMULA FOR THE HARMONIC MEAN	5	4	3	2	1
9. THALES' CALCULATION OF THE HEIGHT OF THE GREAT PYRAMID	5	4	3	2	1
10. ERATOSTHENES' METHOD FOR FINDING THE CIRCUMFERENCE OF THE EARTH	5	4	3	2	1
11. THE EULER LINE (the line through the orthocenter, and circumcenter of a triangle)	5	4	3	2	1
12. VARIOUS PROOFS OF THE PYTHAGOREAN THEOREM	5	4	3	2	1

### **PART II - PRACTICE**

**Directions:** For each of the items in Part I, circle those areas of study which are part of the Geometry course(s) which you teach.

NOTATION: 1 2

ETYMOLOGIES: 1 2

MEASUREMENT: 1 2

MATHEMATICIANS: 1 2 3 4 5 6 7 8 9 10 11 12

### **PART III - COMMENTS**

**Directions:** Please write your comments/questions below. You may include ways that you use the history of mathematics and/or envision its use.

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Thank you for your time and effort!

**APPENDIX B**  
**NAMES OF HIGH SCHOOLS**  
**CONTACTED FOR THE**  
**HISTORY OF MATHEMATICS IN THE CLASSROOM QUESTIONNAIRE**

**Pennsylvania high schools contacted for participation in the History of Mathematics in the Classroom Questionnaire:**

Abington High School	Lower Merion High School
Academy Park High School	Methacton High School
Avon Grove High School	Morrisville High School
Bensalem High School	Neshaminy High School
Bristol High School	Owen J. Roberts High School
Central Bucks East High School	Penn Wood High School
Central Bucks West High School	Perkiomen Valley High School
Chester High School	Phoenixville Area High School
Chichester High School	Plymouth-Whitemarsh High School
Coatesville Area High School	Quakertown Community High School
Council Rock High School	Ridley High School
Downingtown High School	Souderton Area High School
Garnet Valley High School	Strath Haven High School
Great Valley High School	Sun Valley High School
Harry S. Truman High School	Unionville High School
Hatboro-Horsham High School	Upper Darby High School
Haverford Township High School	Upper Merion High School
Henderson High School	Upper Perkiomen High School
Interboro High School	William Tennent High School
Jenkintown High School	Wissahickon High School

**APPENDIX C**  
**RESPONSE TOTALS FOR EACH AREA IN**  
**HISTORY OF MATHEMATICS IN THE CLASSROOM QUESTIONNAIRE**



## Questionnaire: History of Mathematics in the Classroom

### PART I: OPINION

**Directions:** The following is a list of areas in which the history of mathematics may be an integral part of a lesson. Please respond to the importance of each area of study by circling the appropriate number according to the scale below.

#### Scale

- 5 - Very Important
- 4 - Important
- 3 - Somewhat Important
- 2 - Of Little Importance
- 1 - Not Important

NOTATION	5	4	3	2	1
1. THE ORIGIN OF GEOMETRIC SYMBOLS AND NOTATION	2	13	21	12	2
2. USES FOR THE GREEK ALPHABET	2	12	20	12	4
<b>ETYMOLOGIES (WORD ORIGINS)</b>					
1. CONNECTIONS BETWEEN WORD PARTS AND THE ENGLISH DEFINITION (e.g. isos- (equal); skelos (legs) for isosceles)	9	22	14	5	0
2. LATIN OR GREEK PREFIXES AND ROOTS (e.g. circum- (around), scribe (to write) are Latin roots for circumscribe)	9	25	12	6	0
<b>MEASUREMENT</b>					
1. INDIRECT MEASUREMENT	6	18	16	7	2
2. SURVEYING INSTRUMENTS	3	8	23	11	5
<b>MATHEMATICIANS</b>					
1. PICTURES OF MATHEMATICIANS	1	12	19	13	5
2. BIOGRAPHIES	2	14	18	14	2
3. ANECDOTES	6	13	21	7	2
4. <u>EUCLID'S ELEMENTS</u> AS THE FOUNDATION FOR GEOMETRY AS AN AXIOMATIC SYSTEM	11	15	19	2	3

Next Page ----->

5. EUCLID'S USE OF THE REDUCTIO AD ABSURDUM PROOF	<u>5</u>	<u>4</u>	<u>3</u>	<u>2</u>	<u>1</u>
	3	4	20	12	9
6. ARCHIMEDES' CALCULATION OF $\pi$	3	13	18	11	4
7. DÉSCARTES' DEVELOPMENT OF A COORDINATE SYSTEM	3	27	12	6	1
8. PYTHAGORAS' FORMULA FOR THE HARMONIC MEAN	2	11	18	12	5
9. THALES' CALCULATION OF THE HEIGHT OF THE GREAT PYRAMID	6	9	17	14	3
10. ERATOSTHENES' METHOD FOR FINDING THE CIRCUMFERENCE OF THE EARTH	6	11	17	12	3
11. THE EULER LINE (the line through the centroid, orthocenter and circumcenter of a triangle)	0	8	21	15	3
12. VARIOUS PROOFS OF THE PYTHAGOREAN THEOREM	10	14	20	4	1

### **PART II - PRACTICE**

**Directions:** For each of the items in Part I, circle those areas of study which are part of the Geometry course(s) which you teach.

NOTATION: 20 19

ETYMOLOGIES: 38 37

MEASUREMENT: 26 8

MATHEMATICIANS: 17 20 30 31 12 15 26 10 11 11 14 26

### **PART III - COMMENTS**

**Directions:** Please write your comments/questions below. You may include ways that you use the history of mathematics and/or envision its use.

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Thank you for your time and effort!