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### Stock Loan Lotteries and Individual Investment Performance

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# Stock loan lotteries and individual investor performance

Jordan Moore <sup>1</sup>  
January 16, 2017

## Abstract

Individual investors trade excessively, sell winners too soon, and overweight stocks with lottery features and low expected returns. This paper models a financial innovation to address these biases and improve individual investor performance. Individual investors pledge shares of stock to an exchange for multiple periods and face a steep penalty for redeeming shares early. The exchange lends the shares to institutions and holds a lottery with the lending fees. I extend the Barberis and Xiong (2009) discrete-time model of realization utility to include stock loan lotteries. Investors with cumulative prospect theory preferences are reluctant to forgo trading opportunities for fixed stock loan fees, but are far more willing to forgo trading opportunities for stock loan lottery tickets. For a wide range of feasible parameter values, introducing stock loan lotteries provides individual investors both increased utility and increased expected wealth. Stock loan lotteries provide greater utility to poorer investors, who typically exhibit stronger lottery preferences. Introducing transactions costs, leverage constraints, and taxes strengthens the benefits of stock loan lotteries.

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## 1. Introduction

Individual investors trade frequently and hold undiversified portfolios, so they earn lower returns after controlling for risk and transactions costs. What would it take to motivate individual investors to modify their costly behavior? I propose and model a financial innovation to improve the trading performance of individual investors. Barber and Odean (2000) find that individual investors trade excessively and there is a strong negative relation between individual investor performance and trading frequency.<sup>2</sup> French (2008) estimates that active trading costs US equity investors 67 basis points annually. Odean (1998) shows that the disposition effect—the tendency to sell winning positions and realize a gain, reduces the average returns of individual investors. Barber et al. (2009) analyze the complete record of Taiwanese equity trading activity over four years and find that stocks individual investors sell outperform stocks individual investors purchase by 3.8% over the next year. Several studies document a more pronounced disposition effect for investors with less experience and education.<sup>3</sup>

Individual investors tend to hold portfolios with a small number of individual stocks, hoping to earn extraordinary returns. Kumar (2009) finds that individual investors overweight stocks with “lottery features” including low share prices, high volatility and high skew. However, these lottery stocks earn poor returns relative to the market. Investors who are poor, inexperienced, and uneducated are especially likely to overweight lottery stocks. Dorn et al. (2014) provide evidence that when lottery jackpots increase in a particular state or country, trading activity by individual investors in the same state or country falls. Likewise, Barber et al. (2009) document a substantial reduction in the trading of Taiwanese equities following the introduction of legalized gambling. If investors treat

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<sup>2</sup>In data from a large discount broker between 1991 and 1996, the average household turns over more than 75% of its portfolio annually.

<sup>3</sup>These include Feng and Seasholes (2005), Dhar and Zhu (2006), and Frazzini (2006).

lotteries and financial gambling as substitutes, then investors might accept lottery tickets as compensation for not gambling in the financial markets. I test this idea within the Barberis and Xiong (2009) discrete-time model of realization utility. Investors are paid, in the form of lottery tickets, to hold positions for multiple periods. Kearney et al. (2010) document the widespread demand for savings accounts and fixed income investments that incorporate lottery payoffs.

In the Barberis and Xiong (2009) model, investors only experience utility from realizing gains and losses. Investors have Tversky and Kahneman (1992) preferences, so they value gains and losses instead of wealth, react asymmetrically to gains and losses, and evaluate prospects using subjective “decision weights” rather than objective probabilities. Barberis and Xiong (2009) find that these investors aspire to realize gains over multiple episodes, resulting in a disposition effect. However, this tendency to reduce positions in assets with positive excess returns leads to lower expected wealth.

Stock loan lotteries provide a potential solution. An individual investor and a centralized exchange enter into a contract. The investor agrees to hold stocks in his portfolio for multiple periods, and selling stocks early results in a severe penalty.<sup>4</sup> This contract is analogous to a Certificate of Deposit (CD) or a Guaranteed Investment Certificate (GIC). The exchange operates a competitive marketplace in securities lending. When the exchange lends shares, it allocates the fees, net of expenses and profits, to the lottery account of the individual investor. Periodically, the exchange holds a lottery and pays the winner the entire pool of stock lending fees. I find that investors with Tversky and Kahneman (1992) preferences are reluctant to forgo frequent trading for stock loan fees. However, these investors are willing to forgo something they overvalue, frequent trading, for something else they overvalue, lottery tickets. In the data, poor investors have especially strong lottery

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<sup>4</sup>Throughout this paper, I assume that selling stocks early is impossible under the terms of the contract.

preferences. Since stock loan lotteries provide poorer investors a smaller probability of winning a larger payoff, they effectively target heterogeneous preferences among individual investors. Furthermore, stock loan lotteries become more appealing after introducing real market frictions such as transactions costs, leverage constraints, and taxes.

The remainder of this paper is organized as follows. Section 2 compares allocations and outcomes for three models of realization utility: the Barberis and Xiong (2009) model, a model with stock loan fees, and a model with stock loan lotteries. This section assumes that investors use objective probabilities to determine optimal allocations. Section 3 extends the models to incorporate subjective decision weights and defines conditions where individual investors have unconditionally greater welfare, meaning greater expected utility and greater expected wealth. This section also discusses the model implications for heterogeneous investors and market frictions. Section 4 concludes.

## **2. Models of Realization Utility with Objective Probabilities**

This section evaluates investor allocations and outcomes in three models of realization utility, using objective probabilities to calculate expected utility. First, I describe the two-period model in Barberis and Xiong (2009) and replicate their results. Next, I construct a model where investors receive fixed stock loan fees if they commit to holding shares for two periods. Finally, I construct a model where investors receive stock loan lottery tickets if they commit to holding shares for two periods.

### **2.1. Baseline Two-Period Model of Realization Utility**

There are three dates in the model:  $t = 0$ ,  $t = 1$ , and  $t = 2$ . The duration of two periods is calibrated to span one year. There is a representative investor who can invest in two securities. The risk-free asset has a gross return of  $R_f = 1$ . The risky asset has an expected annual gross return of  $\mu$  and annual standard deviation of  $\sigma$ . If one year consists of two

periods, the gross one-period return of the risky asset is modeled as following a binomial distribution:

$$\begin{cases} R_t = R_u = \mu^{0.5} + ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & \pi = 0.5 \\ R_t = R_d = \mu^{0.5} - ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & 1 - \pi = 0.5 \end{cases}$$

$R_u$  and  $R_d$  are gross risky-asset returns in the up and down states,  $\pi$  is the probability of realizing the up state in a particular period, and  $R_t$  is i.i.d. across periods.

The investor maximizes expected cumulative prospect theory utility where the value function, defined over gains and losses, follows the Tversky and Kahneman (1992) functional form:

$$\begin{cases} v(x) = x^\alpha & x \geq 0 \\ v(x) = -\lambda(-x)^\beta & x < 0 \end{cases}$$

The investor with prospect theory preferences is risk averse over gains and risk seeking over losses, so  $\alpha$  and  $\beta$  are restricted to the interval  $(0, 1)$ . Also, the investor is more sensitive to losses, so  $\lambda > 1$ .

The investor is endowed with initial wealth  $W_0$ . At  $t = 0$ , the investor chooses to purchase  $x_0$  shares of the risky asset at a price of  $P_0$  per share. The investor is not allowed to have negative wealth at  $t = 1$ , so  $x_0$  is restricted to the interval:  $[0, \frac{W_0}{P_0 * (1 - R_d)}]$ . The remainder of the investor's wealth is allocated to the risk-free asset. Therefore, at  $t = 1$ , the investor's wealth is distributed:

$$\begin{cases} W_u = W_0 + P_0 X_0 (R_u - 1) & \pi = 0.5 \\ W_d = W_0 + P_0 X_0 (R_d - 1) & 1 - \pi = 0.5 \end{cases}$$

Table 1 summarizes the parameters for the Barberis and Xiong (2009) two-period model of realization utility. At  $t = 1$ , the investor chooses  $x_u$  and  $x_d$ , state-contingent positions in the risky asset. If  $x_u < x_0$  or  $x_d < x_0$ , the investor sells some or all of his initial position, realizing a gain or loss. The investor experiences a burst of prospect theory utility, its magnitude defined by the value function applied to the realized gain or loss. Since the investor cannot have negative wealth at  $t = 2$ , there are two state-specific nonnegativity constraints:  $x_u$  is restricted to the interval:  $[0, \frac{W_u}{P_0 * R_u * (1 - R_d)}]$  and  $x_d$  is restricted to the interval:  $[0, \frac{W_d}{P_0 * R_d * (1 - R_u)}]$ . These maximum allocations depend on both the current stock price and, through the investor's wealth, his  $t = 0$  allocation to the risky asset. At  $t = 2$ , the investor liquidates his position in the risky asset and experiences a second burst of prospect theory utility.

The investor chooses  $x_0$ ,  $x_u$ , and  $x_d$  to maximize expected prospect theory utility:

$$\max_{x_0, x_u, x_d} E_0[v((x_0 - x_1)(P_1 - P_0)) * 1_{x_1 < x_0} + v(x_1 * (P_2 - P_b)) * 1_{x_1 > 0}]$$

The first term measures the investor's realization utility at  $t = 1$ , while the second term measures the investor's realization utility at  $t = 2$ .  $P_b$  is the investor's cost basis, the reference price for evaluating gains and losses at  $t = 2$ . The cost basis depends on whether the investor purchases shares at  $t = 1$  and whether the purchase follows an up state or down state:

$$\begin{cases} P_{bu} = P_{bd} = P_0 & x_1 \leq x_0 \\ P_{bu} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_u}{x_1} & x_1 > x_0 \\ P_{bd} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_d}{x_1} & x_1 > x_0 \end{cases}$$

The value function at each of the four possible  $t = 2$  outcomes ( $uu, ud, du, dd$ ) can be written in terms of the choice variables:

$$v_{uu}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{uu} - P_{bu})) * 1_{x_u > 0}$$

$$v_{ud}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{ud} - P_{bu})) * 1_{x_u > 0}$$

$$v_{du}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{du} - P_{bd})) * 1_{x_d > 0}$$

$$v_{dd}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{dd} - P_{bd})) * 1_{x_d > 0}$$

Because  $\pi = 0.5$ , each node of the binomial tree is equally likely, and the investor maximizes the average value associated with each outcome:

$$\max_{x_0, x_u, x_d} 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

I solve the model numerically by calculating  $E_0(v)$  for all feasible values of  $(x_0, x_u, x_d)$  and choosing arguments that maximize the value function. Table 2 summarizes the model solutions for different values of  $\mu$ , holding all other parameter values constant. When the expected gross annual return of the risky asset is below 1.08, the investor's optimal decision is to invest all his wealth in the risk-free asset. Because the gross return of the risk-free asset is calibrated to  $R_f = 1$ , this conservative investment strategy guarantees  $E_0(v) = 0$ .<sup>5</sup> Because prospect theory investors are more sensitive to losses than gains, they require a substantial risk premium to invest in the risky asset.

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<sup>5</sup>Setting  $R_f = 1$  assumes investors evaluate the performance of the risky investment relative to the risk-free rate instead of relative to 0.



For expected returns between 1.09 and 1.11, the representative investor exhibits a disposition effect. He chooses to invest some of his wealth in the risky asset at  $t = 0$  and takes some profits at  $t = 1$  when the up state occurs. Because the investor has concave realization utility over gains, he prefers to experience gains over multiple bursts. Once the risk premium exceeds 12%, the investor prefers to increase his initial investment following the realization of the up state. Because the investor's portfolio appreciates in the first period, he is able to take more risk before exhausting the nonnegative wealth constraint. For assets with sufficiently high expected returns, the marginal expected prospect theory utility of increasing expected gains in the second period exceeds the marginal utility of realizing gains at  $t = 1$ .

## 2.2. Two-Period Model of Realization Utility with Fixed Stock Loan Fees

In this model, the investor can only choose to purchase shares of the risky asset at  $t = 0$  and promise to hold the position until  $t = 2$ . This commitment allows the centralized exchange to lend shares to institutions who want to short sell the stock. The exchange retains some proportion of the securities lending proceeds as a commission and pays the remainder to the investor at  $t = 2$ .

Since the investor cannot trade at  $t = 1$ , there is a single choice variable,  $x_0$ . The one new parameter in this model is  $f$ , the lending fee. I consider two values for  $f$ : five basis points (0.0005), a realistic fee for US large-cap equities, and 50 basis points (0.005), a realistic fee for US small-cap equities or foreign equities. These parameter estimates are within the range of stock lending fees in the D'Avolio (2002) and Cohen et al. (2003) data.

In this model, the investor has a single non-negative wealth constraint at  $t = 2$ . This means  $x_0$  is restricted to the interval:  $[0, \frac{W_0}{P_0*(1-R_d^2-f)}]$ . This constraint ensures that if the down state occurs in both periods, the investor's initial wealth and the stock loan fees he

receives at  $t = 2$  will exactly cover his losses. In this model, the investor maximizes:

$$\max_{x_0} E_0(v) = 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

The value at each possible  $t = 2$  outcome is:

$$v_{uu}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{uu} - P_0)))$$

$$v_{ud}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{ud} - P_0)))$$

$$v_{du}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{du} - P_0)))$$

$$v_{dd}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{dd} - P_0)))$$

In this model, the best outcome is always a corner solution. If the prospective two-period gamble has positive expected utility for some positive allocation, then the gamble has strictly greater expected utility for a larger positive allocation. Therefore, the investor will either invest fully in the risky asset, exhausting the nonnegative wealth constraint, or invest fully in the risk-free asset.

**Proposition 1:** In the two-period model of realization utility with fixed stock loan fees, the investor either invests fully in the risky asset or invests fully in the risk-free asset.

**Proof:** See Appendix

Table 3 summarizes the investor's optimal allocations and outcomes in the two-period model with fixed stock loan fees. In the absence of any stock loan fees, the investor is worse off by choosing only among allocations with no rebalancing ( $x_0 = x_u = x_d$ ). All the feasible allocations in the model with stock loan fees are feasible in the baseline model, but the converse is not true. Thus, the investor is maximizing expected utility over a strictly smaller choice set. Stock loan fees of five basis points ( $f = 5$ ) do not sufficiently compensate the investor for losing the option to trade at  $t = 1$ . As the center panel of

Table 3 shows, for all values of  $\mu$ , the investor's best outcome with stock loan fees of five basis points is worse than the investor's best outcome in the baseline model.

On the other hand, for a stock loan fee of 50 basis points ( $f = 50$ ), the investor's best outcome improves for the same range of risky asset returns that generate a disposition effect in Barberis and Xiong (2009). The desire to realize gains over multiple periods is costly. Given that returns are i.i.d. across periods and future utility is not discounted, an investor who prefers a risky asset investment at  $t = 0$  should prefer the same risky asset investment at  $t = 1$ . To the extent that the investor forgoes expected future wealth for realization utility, sufficiently large stock loan fees can be an effective deterrent.

### 2.3. Two-Period Model of Realization Utility with Stock Loan Lotteries

In this model, the investor still promises to hold any shares he purchases at  $t = 0$  until  $t = 2$ . However, instead of receiving a stock loan fee ( $f * P_0 * x_0$ ) at  $t = 2$ , the investor receives stock loan lottery tickets. Periodically, the exchange holds a lottery and pays the winner the entire pool of stock loan fees. The stock loan fees are net of the expenses and profits of the exchange and the lottery itself is actuarially fair. In Kahneman and Tversky (1979) notation, a single stock loan lottery at  $t = 2$  is a gamble of  $(\frac{f * P_0 * x_0}{p}, p; 0, 1 - p)$ , where  $p$  is the probability of winning the lottery.<sup>6</sup> In this model, the parameters provide three sources of variation. First, as in section 2.2, the stock loan fee ( $f$ ) is either five basis points or 50 basis points. Second,  $p$  is either 0.01 or 0.1. Third, there is either a single lottery at  $t = 2$  for all of the loan fees, or two lotteries at  $t = 1$  and  $t = 2$ , each for half of the loan fees. Each of these lotteries is equivalent to the Kahneman and Tversky (1979) gamble:  $(\frac{0.5 * f * P_0 * x_0}{p}, p; 0, 1 - p)$ .

When there is a single lottery at  $t = 2$ , the investor maximizes:

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<sup>6</sup>I assume the investor references the gamble to the baseline model instead of the fees model.

$$\max_{x_0} E_0(v) = 0.25 * [p * (v_{uuw} + v_{udw} + v_{duw} + v_{ddw}) + (1-p) * (v_{uul} + v_{udl} + v_{dul} + v_{ddl})]$$

In this notation,  $w$  is the state where the investor wins the stock loan lottery and  $l$  is the state where the investor loses the stock loan lottery. The values in the two outcomes following the realization of two up states are:

$$v_{uuw}(x_0) = v\left(\left(\frac{f * p_0 * x_0}{p} + x_0 * (P_{uu} - P_0)\right)\right)$$

$$v_{uul}(x_0) = v(x_0 * (P_{uu} - P_0))$$

In the contingent value formulas for the six other outcomes ( $udw, udl, duw, dul, ddw, ddl$ ), the only difference is the share price after two periods, which is either  $P_{ud}, P_{du}$ , or  $P_{dd}$ . In the version of the model with lotteries at  $t = 1$  and  $t = 2$ , there are 16 possible outcomes, depending on whether the up or down state is realized in each period, and whether the investor wins or loses the two lotteries. The investor maximizes:

$$\max_{x_0} 0.25 * [p^2 * (v_{uuww} + v_{udww} + v_{duww} + v_{ddww}) + p(1-p) * (v_{uuwl} + v_{udwl} + v_{duwl} + v_{ddwl} + v_{uulw} + v_{udlw} + v_{dulw} + v_{ddlw}) + (1-p)^2 * (v_{uull} + v_{udll} + v_{dull} + v_{dull})]$$

Because the investor savors each burst of prospect theory utility distinctly, a lone lottery win at  $t = 1$  by itself has a different value than a lone lottery win at  $t = 2$  accompanied by realized gains or losses. The four representative value formulas are:

$$v_{uuww}(x_0) = v\left(\frac{0.5f * p_0 * x_0}{p}\right) + v\left(\frac{0.5f * p_0 * x_0}{p} + x_0 * (P_{uu} - P_0)\right)$$

$$v_{uuwl}(x_0) = v\left(\frac{0.5f * p_0 * x_0}{p}\right) + v(x_0 * (P_{uu} - P_0))$$

$$v_{uulw}(x_0) = v\left(\frac{0.5f * p_0 * x_0}{p} + x_0 * (P_{uu} - P_0)\right)$$

$$v_{uull}(x_0) = v(x_0 * (P_{uu} - P_0))$$

Whether there are one or two lotteries, the non-negative wealth constraint requires that  $x_0$  is restricted to the interval:  $[0, \frac{W_0}{P_0 * (1 - R_d^2)}]$ . The worst case scenario is that both periods

are down states and the investor doesn't win any lotteries. In versions of the model with stock loan lotteries, the optimal allocation is always a corner solution. If the expected return is below some threshold, the investor keeps all his wealth in the risk-free asset. If the expected return is above the threshold, the investor exhausts the nonnegative wealth constraint.

Table 4 summarizes the investor's best outcomes in each version of the model with stock loan lotteries. The investor always prefers earning a fixed stock loan fee to participating in a risky stock loan lottery with the same expected value. Furthermore, holding the expected value of the lottery constant, the investor always prefers the lottery with  $p = 0.1$  to the lottery with  $p = 0.01$ . Finally, the investor always prefers participating in two smaller lotteries at  $t = 1$  and  $t = 2$  to one larger lottery at  $t = 2$ . These results follow from the concavity of the prospect theory value function over gains. Of course, the investor always prefers receiving a stock loan fee of 50 basis points to receiving a fee of five basis points.

### **3. Model Extensions**

This section develops the two-period model of realization utility with stock loan lotteries. Following Tversky and Kahneman (1992), investors maximize expected utility using decision weights, rather than objective probabilities. Investors are willing to pay more than the actuarially fair price to participate in lotteries, suggesting that introducing lotteries increases the maximum expected utility. Investors who enter into stock loan lottery contracts do not realize gains prematurely and therefore can also earn higher returns. In an economy with many investors, those with different levels of wealth will have different Kahneman and Tversky (1979) gambles in the lottery. Specifically, poorer investors, who hold lottery stocks in larger proportions in the data, have larger exposure to lottery features.

Finally, market frictions increase the appeal of lotteries by increasing the motivation for investors to refrain from excessive trading.

### 3.1. Decision Weights

Tversky and Kahneman (1992) use experimental data to estimate a functional form for  $w(p)$ , the “weighting function” that converts objective probabilities into decision weights. A lottery is a gamble of the form  $(1,p;0,1-p)$ .<sup>7</sup> For nonnegative gambles, Tversky and Kahneman (1992) estimate the weighting function as a two-part power function:

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

Likewise, for nonpositive gambles, the weighting function is:

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$$

Using experimental data, Tversky and Kahneman (1992) estimate  $\gamma = 0.61$  and  $\delta = 0.69$ . These values imply that investors overvalue gambles, both positive and negative, with low probabilities of success. Although the experimental evidence in Tversky and Kahneman (1992) uses small wagers, Kachelmeier and Shehata (1992) show that for large wagers, risk seeking over small probabilities holds for both positive and negative rewards.<sup>8</sup>

Figure 1 shows the potential for stock loan lotteries to increase the investor’s expected utility. The exchange offering stock loan lotteries has monopoly power because they hold the investor’s stock loan fees and can offer them denominated in cash or lottery tickets. For a given lottery, a proxy for the potential increase in utility is  $\max[w^+(p) - p, 0]$ , where  $p$  is the probability of winning. When prospect theory investors overvalue the probability of winning the lottery, the monopolist can offer the lottery at the actuarially fair price

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<sup>7</sup>The investor’s reference point is receiving a free lottery ticket.

<sup>8</sup>The authors offer lotteries in China, where the amount of the wager comprises a substantial portion of the subject’s income.

implied by the decision weights and extract all the rents. When prospect theory investors undervalue the probability of winning the lottery, there are no profit opportunities, so the monopolist doesn't offer the lottery.<sup>9</sup> According to Figure 1, individuals overvalue all lotteries with  $p \leq 0.338$ . The exchange has maximum potential profit opportunities from offering lotteries with win probabilities from 0.07 to 0.11, which investors overvalue by 8-9%.

Table 5 summarizes the best investor outcomes in two-period models of realization utility, calculating expected value using both objective probabilities and decision weights. I consider the 11 specifications of the model from Section 2. Specification 1 is the Barberis and Xiong (2009) baseline model (B). Specifications 2 and 3 are models with fixed loan fees (F). The investor chooses a single risky asset allocation at  $t = 0$  ( $x_0$ ) and cannot trade shares at  $t = 1$ . The investor receives a fee ( $f$ ) for allowing the exchange to lend shares between  $t = 0$  and  $t = 2$ . The remaining models include stock loan lotteries. In specifications 4 through 7, there is a single lottery at  $t = 2$  and the investor has probability  $p$  of winning  $f * P_0 * x_0 / p$ . In specifications 8 through 11, there are lotteries at  $t = 1$  and  $t = 2$  and the investor has probability  $p$  of winning  $0.5 * f * P_0 * x_0 / p$  in each lottery. For each specification, the top panel presents the investor's maximum expected prospect theory utility calculated with objective probabilities. The bottom panel presents the investor's maximum expected prospect theory utility calculated with decision weights.

For all specifications with stock loan lotteries, the investor's maximum expected utility is greater when prospect theory utility is calculated with decision weights rather than with objective probabilities. The investor's best outcome with stock loan lotteries is better than the investor's best outcome with fixed stock loan fees if utility is calculated with decision weights, rather than similar or worse if utility is calculated with objective probabilities. The

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<sup>9</sup>Alternatively, in a perfectly competitive lottery marketplace, the investors extract all the utility gains.

fixed stock loan fees increase gains or decrease losses by a marginal amount with certainty. On the other hand, stock loan lotteries increase gains or decrease losses dramatically for a small proportion of outcomes that investors substantially overvalue. As a result, relative to the Barberis and Xiong (2009) two-period model, it is much easier to increase expected utility through stock loan lotteries than through stock loan fees.

### 3.2. Conditions for Welfare Gains

Barberis (2013) argues that prospect theory complements traditional economic theory. Individuals care about expected wealth and variability of wealth as well as gains or losses relative to a reference point. In this section, I argue that when two conditions are satisfied, introducing stock loan lotteries yields unconditional welfare gains. First, the investor must experience greater expected prospect theory utility in a model with stock loan lotteries than he experiences in the baseline model. The expected prospect theory utility calculation should use decision weights instead of objective probabilities because utility is a subjective measure of the investor's happiness:

$$E_0^*(v_L) > E_0^*(v_B)$$

Second, if the investor maximizes  $E[U(W_2)]$  and has risk-neutral preferences, he must have higher expected wealth at  $t = 2$  in the model with stock loan lotteries than he has in the baseline model:<sup>10</sup>

$$E_0^*[U(W_{2,L})] > E_0^*[U(W_{2,B})]$$

The expected wealth calculation should use objective probabilities to measure the investor's actual financial position at  $t = 2$ . In the baseline model:

$$E_0^*[U(W_{2,B})] = W_0 + P_0 * (0.5 * x_{0,B}^* + 0.25 * x_{u,B}^* + 0.25 * x_{d,B}^*) * (\mu - 1)$$

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<sup>10</sup>Prospect theory investors are risk averse over gains and risk seeking over losses. The assumption of risk neutrality splits the difference.



In the model with stock loan lotteries:

$$E_0^*[U(W_{2,L})] = W_0 + (P_0 * x_{0,L}^*) * (\mu + f - 1)$$

Figure 2 presents conditions in which introducing stock loan lotteries delivers unconditional welfare gains. For all points above the solid curve, investors in the model with stock loan lotteries have strictly higher welfare than investors in the baseline model. In other words, for these points in  $(\mu, f)$  space, the utility-maximizing allocation in the model with lotteries produces greater expected utility and greater expected wealth than the utility-maximizing allocation in the baseline model. The solid curve is U-shaped, first decreasing in  $\mu$ , then increasing in  $\mu$ . As the Sharpe ratio of the risky asset increases from low to moderate, the lottery discourages the investor from taking profits and destroying wealth. As the Sharpe ratio of the risky asset increases from moderate to high, the lottery keeps the investor from increasing his position in the risky asset at  $t = 1$  to exhaust the relaxed non-negative wealth constraint. For all points above the dotted curve in  $(\mu, f)$  space, investors in the model with stock loan fees have strictly higher welfare than investors in the baseline model. The dotted curve is also U-shaped, but for a wide range of expected asset returns, introducing stock loan lotteries unconditionally improves investor welfare, while introducing stock loan fees does not. This area is denoted by the shaded region.

### 3.3. Implications for Equilibrium with Heterogeneous Agents

In equilibrium, there are many individual investors. All investors benefit from the lottery because the probability of any individual investor winning the lottery is in the region where the decision weight exceeds the objective probability. Individual investors have variation in risky asset holdings because of variation in wealth, and a number of studies suggest that poor individual investors exhibit especially strong biases. For instance, Dhar and Zhu (2006) show that the trading activity of poor investors shows a larger disposition effect,

while Kumar (2009) shows that poor investors hold a larger proportion of “lottery stocks” in their portfolios.

Consider a stock loan marketplace with many individual investors, and each investor,  $i$ , owns  $X_{0,i}$  shares of the risky asset at  $t = 0$ . If there is a single lottery at  $t = 2$  with a single winner, then each investor’s probability of winning the lottery depends on the ratio of his allocation to the aggregate allocation of all investors,  $X_{0,A}$ . Since investors who buy the risky asset always exhaust the non-negative wealth constraint, each investor’s probability of winning the lottery is also equivalent to his wealth share:

$$p = \frac{X_{0,i}}{X_{0,A}} = \frac{W_{0,i}}{W_{0,A}}$$

If each investor has risk-neutral preferences, then expected utility is proportional to initial wealth:

$$E[U(W_{2,i})] = E(W_{2,i}) = (\mu + f) * W_{0,i}$$

One way to examine the relative appeal of stock loan lotteries to different investors is to see how a standardized measure of utility varies according to wealth share. Define  $\Pi$  as this standardized measure of utility per unit of wealth:

$$\Pi_i = \frac{E[U(W_{2,i})|p,v(),w^+(p)]}{W_{0,i}}$$

Figure 3 shows how standardized utility varies according to wealth share in four different models of realization utility. The solid lines show standardized utility for models with a single stock loan lottery at  $t = 2$  and stock loan fees of five and 50 basis points. The dotted lines show standardized utility for models with stock loan fees of five and 50 basis points. For all models, the annualized expected gross return of the risky asset ( $\mu$ ), is 1.12. This risky asset return is sufficient for all investors to exhaust the non-negative wealth constraint. In all four models, standardized utility is strictly downward sloping with respect to wealth

share. In other words, these models are regressive in the sense that poor investors benefit disproportionately in utility terms.

Models with stock loan fees are regressive because the prospect theory value function is concave over gains, so the marginal utility of wealth is strictly decreasing with increasing wealth. The concavity of the value function implies that increasing loan fees also benefits the poorest investors disproportionately. The economy with stock loan lotteries is substantially more regressive. This follows from the functional form of the decision weighting function. The ratio  $w^+(p)/p$  is strictly decreasing with increasing  $p$ . The poorest investors have the lowest probabilities of winning the lotteries and place the highest value on the lotteries relative to the value implied by objective probabilities. For this reason, the regressive effect of increasing  $f$  is greater in the model with stock loan lotteries than in the model with stock loan fees.

### **3.4. Market Frictions: Transactions Costs, Leverage, and Taxes**

In all three models of realization utility, I assume markets are frictionless. However, excessive trading by individual investors is a problem, in part, because trading is costly. French (2008) estimates that the annual total cost of active trading consistently ranges from 61 to 74 basis points from 1990 to 2006. These constant trading costs are the result of two countervailing trends. As studies such as Novy-Marx and Velikov (2016) document, the cost of trading a share of stock decreases significantly over time. On the other hand, French (2008) and others document a significant upward time trend in share turnover. I model trading costs by introducing a new parameter,  $\rho$ , representing round-trip transactions costs. Since the two periods in the model correspond to a year, and all positions are closed at  $t = 2$ , I follow the French (2008) estimates and set  $\mu = 0.013$ .

Another important market friction is the cost and availability of leverage. The Barberis

and Xiong (2009) model assumes that investors can borrow or lend at the risk-free rate, and the non-negative wealth constraint ultimately determines the maximum leverage. In fact, Frazzini and Pedersen (2014) show that leverage constraints lead to high demand and low expected returns for high-beta assets. Barberis and Xiong (2009) acknowledge that while optimal allocations imply the use of substantial leverage, only a small proportion of individual investors use leverage. Since the US Federal Reserve Board has maintained a 50% initial margin requirement since 1974, I model leverage constraints by limiting the investor's risky asset investment to twice his wealth. In all three models, this restricts  $x_0 \leq \frac{2*W_0}{P_0}$ . In the baseline models, this also restricts the state-contingent investments at  $t = 1$ :  $x_u \leq \frac{2*W_u}{P_u}$  and  $x_d \leq \frac{2*W_d}{P_d}$ .

Figure 4 shows how market frictions change the potential welfare benefits of stock loan lotteries. For different values of  $\mu$ , the annualized expected gross return of the risky asset, I solve all three models in a perfect market as well as a market with trading costs and leverage constraints. For each value of  $\mu$ , in each environment, I calculate  $f_F$  and  $f_L$ , the minimum fee that provides investors in the fee and lottery models unconditionally greater welfare than in the baseline model. For each  $\mu$ , a proxy for the potential welfare benefits of introducing stock loan lotteries is  $max[f_F(\mu) - f_L(\mu), 0]$ . The solid line shows the potential for welfare improvement with market frictions, while the dotted line shows the potential for welfare improvement with perfect markets. Market frictions increase the potential for lotteries to improve welfare for two reasons. First, trading costs reduce the effective return of all risky investments. As Figure 2 shows, it is far easier to persuade investors to buy assets with low Sharpe ratios by using stock loan lotteries than by using stock loan fees. Second, even when allocations using maximum leverage are lower than those chosen in a model without leverage constraints, investors still typically choose allocations with  $x_u < x_0$  to realize gains spread out over multiple episodes. In a model with capital gains taxes, the

value of stock loan lotteries in discouraging investors from realizing gains prematurely increases further.

#### **4. Conclusion**

The excessive trading of individual investors has large economic consequences. Private firms are motivated to exploit the psychological biases that cause excessive trading. For instance, brokerage firms encourage investors to use mobile apps in order to trade more frequently. Stock loan lotteries have the potential to reduce excessive trading by individual investors. Since individual investors overvalue realizing gains, they need to be compensated to forgo the excessive trading necessary to realize gains. It is much cheaper to compensate individual investors with stock loan lottery tickets than with stock loan fees because individual investors with prospect theory preferences overvalue participation in lotteries. Investors participating in stock loan lotteries can experience greater expected utility and earn higher expected returns, while other market participants still earn profits from securities lending and administering the lottery itself. Two promising features of stock loan lotteries are that they provide the greatest utility to the poorest investors and that the benefits increase as the level of market frictions increase. These results suggest the potential for stock loan lotteries to improve the performance and welfare of individual investors.

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**Table 1: Parameter Values** This table lists the important parameters, symbols, and calibrated values for the Barberis and Xiong (2009) two-period model of realization utility. The investor has  $W_0$  in wealth at  $t = 0$ . He allocates his wealth between the risk-free asset, which has a normalized gross return,  $R_f = 1$ , and a risky asset, with annualized mean return  $\mu$  and annualized standard deviation  $\sigma$ . The risky asset return in each period has a binomial distribution with an equal probability of  $R_u$  return in the up state and  $R_d$  return in the down state, where  $R_u$  and  $R_d$  are chosen to match  $\mu$  and  $\sigma$ . The investor chooses  $x_0$ , the  $t = 0$  risky asset allocation,  $x_u$ , the  $t = 1$  risky asset allocation following the up state, and  $x_d$ , the risky asset allocation following the down state. The investor maximizes  $E_0(v)$ , the current expected value at  $t = 1$  and  $t = 2$ , and  $v()$  is the Tversky and Kahneman (1992) value function applied to realized gains and losses.

Parameter	Symbol	Value(s)
Initial Wealth	$W_0$	40
Risky Asset: Initial Price	$P_0$	40
Risky Asset: Annualized Mean Return	$\mu$	1.05-1.15
Risky Asset: Annualized SD Return	$\sigma$	0.3
Value Function: Concavity	$\alpha$	0.88
Value Function: Convexity	$\beta$	0.88
Value Function: Sensitivity	$\lambda$	2.25



**Table 2: Optimal Allocations in the Baseline Model** For different values of  $\mu$ , the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in the baseline Barberis and Xiong (2009) two-period model of realization utility. In all cases,  $\sigma$ , the annualized standard deviation of risky asset returns, is 0.3. Table 1 lists and describes the other important model parameters. The model assumes that there are two periods in a year, and the gross one-period return of the risky asset is modeled as following a binomial distribution with equal probabilities of realizing a return of  $R_u$  in the up state and  $R_d$  in the down state. The choice variables are the risky asset allocations at  $t = 0$  ( $x_0$ ) and the risky asset allocations at  $t = 1$  following the realization of the up ( $x_u$ ) or down ( $x_d$ ) states.  $E_0(v)$  is expected  $t = 0$  cumulative prospect theory utility. The investor experiences utility at  $t = 1$  and  $t = 2$  if he realizes gains or losses, and the bursts of future prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992).

$\mu$	$R_u$	$R_d$	$x_0^*$	$x_u^*$	$x_d^*$	$E_0^*(v)$
1.05	1.230	0.820	0	0	0	0
1.06	1.234	0.826	0	0	0	0
1.07	1.238	0.831	0	0	0	0
1.08	1.241	0.837	0	0	0	0
1.09	1.245	0.843	3.4	2.6	3.4	0.01
1.10	1.249	0.848	3.6	2.8	3.6	1.09
1.11	1.253	0.854	3.7	3.0	3.7	2.22
1.12	1.257	0.860	3.8	5.4	3.8	3.66
1.13	1.261	0.865	3.9	5.8	3.9	5.19
1.14	1.265	0.871	4.1	6.5	4.2	6.97
1.15	1.269	0.876	4.2	7.2	4.4	8.95

**Table 3: Optimal Allocations in the Model with Stock Loan Fees** For different values of  $\mu$ , the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in various two-period models of realization utility. The left panel shows results from the baseline (B) model. The choice variables are the risky asset allocations at  $t = 0$  ( $x_0$ ) and the risky asset allocations at  $t = 1$  following the realization of the up ( $x_u$ ) or down ( $x_d$ ) states.  $E_0(v)$  is the expected  $t = 0$  cumulative prospect theory utility the investor experiences at  $t = 1$  and  $t = 2$  if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The center and right panels show results from models with stock loan fees (F). The investor chooses a single risky asset allocation at  $t = 0$  ( $x_0$ ) and is not allowed to trade shares at  $t = 1$ . The investor receives a fee of  $f$  for allowing the exchange to lend shares between  $t = 0$  and  $t = 2$ . The center panel shows the investor's optimal allocation and best outcome when  $f$  is five basis points. The right panel shows the investor's optimal allocation and best outcome when  $f$  is 50 basis points. The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan fees is greater than the investor's maximum utility in the baseline model.

Model	B				F(5)		F(50)	
$\mu$	$x_0^*$	$x_u^*$	$x_d^*$	$E_0^*(v)$	$x_0^*$	$E_0^*(v)$	$x_0^*$	$E_0^*(v)$
1.05	0	0	0	0	0	0	0	0
1.06	0	0	0	0	0	0	0	0
1.07	0	0	0	0	0	0	0	0
1.08	0	0	0	0	0	0	0	0
1.09	3.4	2.6	3.4	0.01	0	0	3.5	*0.11
1.10	3.6	2.8	3.6	1.09	3.6	0.73	3.6	*1.25
1.11	3.7	3.0	3.7	2.22	3.7	1.90	3.8	*2.46
1.12	3.8	5.5	3.8	3.65	3.8	3.14	3.9	*3.73
1.13	3.9	5.8	3.9	5.19	4.0	4.46	4.1	5.09
1.14	4.1	6.5	4.2	6.97	4.1	5.86	4.2	6.54
1.15	4.2	7.2	4.4	8.95	4.3	7.37	4.4	8.09

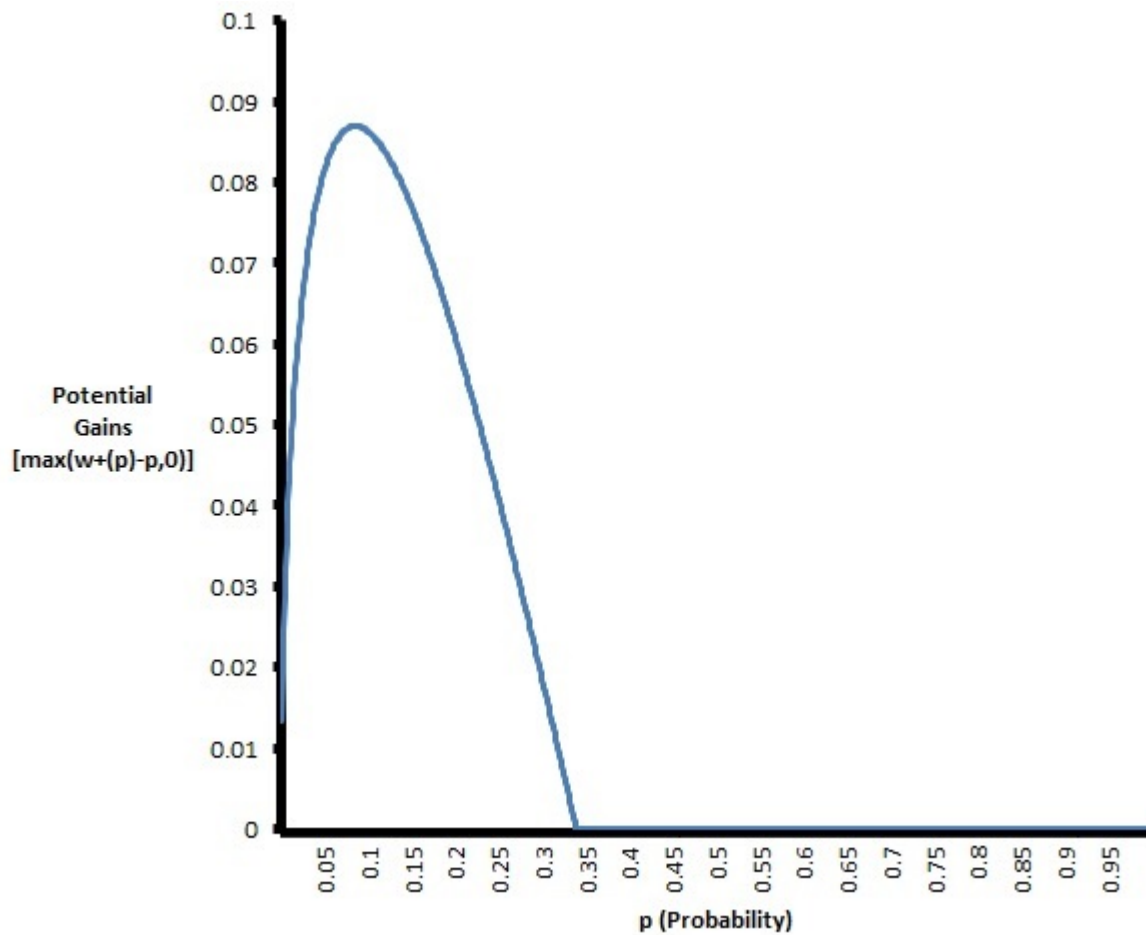
**Table 4: Best Outcomes in the Model with Stock Loan Lotteries** For different values of  $\mu$ , the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility with stock loan lotteries. The only choice variable is the risky asset allocation at  $t = 0$  ( $x_0$ ), and the investor is not allowed to trade these shares at  $t = 1$ . The values in the table are  $E_0^*(v)$ , the expected  $t = 0$  cumulative prospect theory utility the investor experiences at  $t = 1$  and  $t = 2$  if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The investor receives a fee of  $f$  for allowing the exchange to lend shares between  $t = 0$  and  $t = 2$ . The fee is pooled into a lottery, and the investor has probability  $p$  of winning the lottery. In the left panel, there is a single lottery at  $t = 2$  where the investor has probability  $p$  of winning  $f * P_0 * x_0/p$ . In the right panel, there are lotteries at  $t = 1$  and  $t = 2$  and in each lottery, the investor has probability  $p$  of winning  $0.5 * f * P_0 * x_0/p$ . The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan lotteries is greater than the investor's maximum utility in the baseline model.

p	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
f (BPs)	5	5	50	50	5	5	50	50
Model	L(2)	L(2)	L(2)	L(2)	L(1,2)	L(1,2)	L(1,2)	L(1,2)
$\mu = 1.05$	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0	0	0	0
$\mu = 1.09$	0	0	*0.04	*0.10	0	0	*0.06	*0.13
$\mu = 1.10$	0.72	0.73	*1.16	*1.23	0.73	0.74	*1.18	*1.26
$\mu = 1.11$	1.90	1.90	*2.34	*2.41	1.90	1.91	*2.37	*2.44
$\mu = 1.12$	3.14	3.14	3.59	*3.66	3.14	3.15	3.62	*3.70
$\mu = 1.13$	4.45	4.45	4.92	4.99	4.45	4.47	4.95	5.03
$\mu = 1.14$	5.85	5.85	6.33	6.41	5.86	5.87	6.37	6.45
$\mu = 1.15$	7.35	7.35	7.85	7.94	7.36	7.37	7.89	7.97

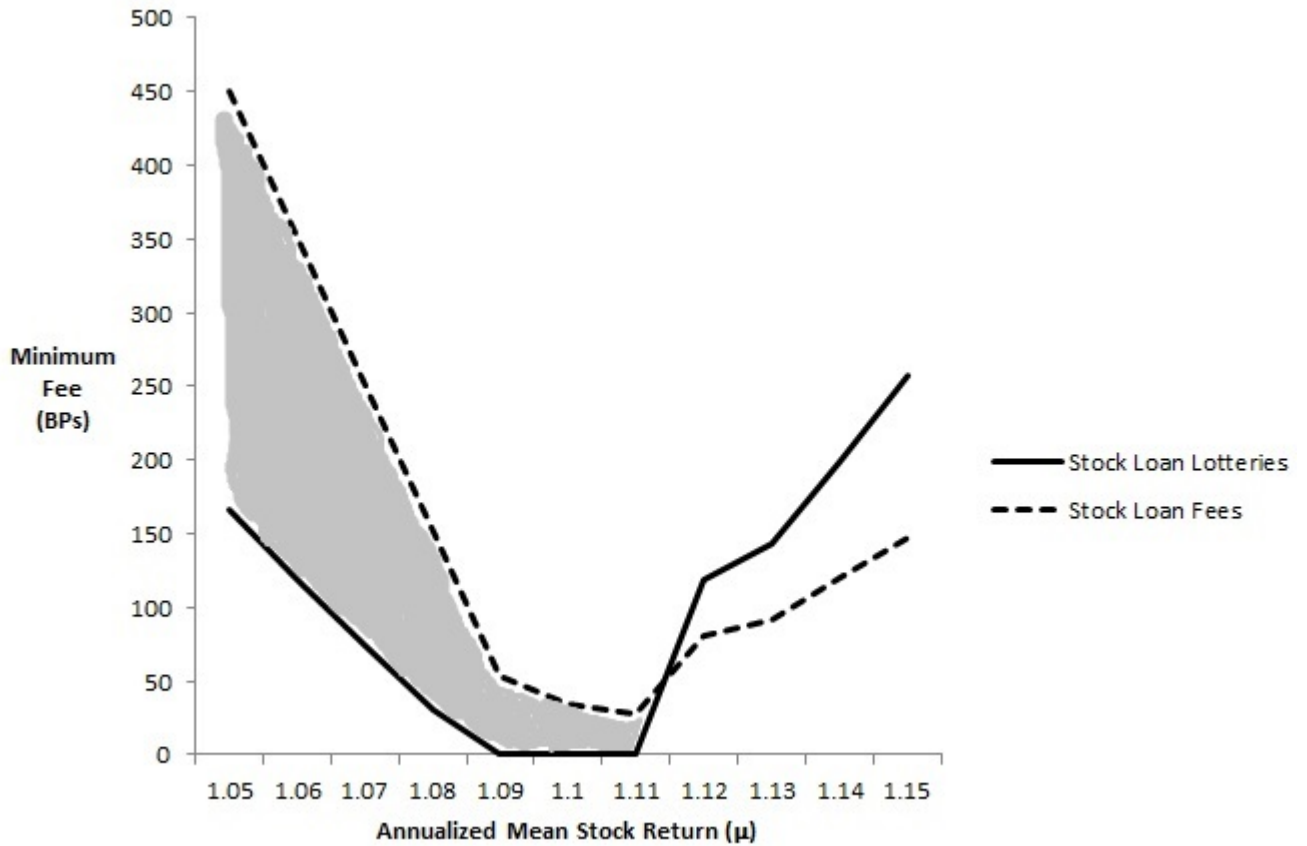
**Table 5: Best Outcomes: Stock Loan Lotteries with Decision Weights** For different values of  $\mu$ , the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility. Specification 1 is the baseline model in Barberis and Xiong (2009). The choice variables are the risky asset allocations at  $t = 0$  ( $x_0$ ) and the risky asset allocations at  $t = 1$  following an up ( $x_u$ ) or down ( $x_d$ ) return in the first period. Specifications 2 and 3 are models where the investor chooses a single risky asset allocation at  $t = 0$  ( $x_0$ ) and is not allowed to trade shares at  $t = 1$ . The investor receives a fee of  $f$  for allowing the exchange to lend shares between  $t = 0$  and  $t = 2$ . Specifications 4 through 11 are models where the stock loan fee is pooled into a lottery, and the investor has probability  $p$  of winning the lottery. In specifications 4 through 7, there is a single lottery (L) at  $t = 2$  where the investor has probability  $p$  of winning  $f * P_0 * x_0 / p$ . In specifications 8 through 11, there are lotteries at  $t = 1$  and  $t = 2$  and in each lottery, the investor has probability  $p$  of winning  $0.5 * f * P_0 * x_0 / p$ . The values in the table are  $E_0(v)$ , the expected  $t = 0$  cumulative prospect theory utility the investor experiences at  $t = 1$  and  $t = 2$  if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). In the top panel, the investor maximizes expected prospect theory utility using objective probabilities. In the bottom panel, the investor maximizes prospect theory utility using decision weights with the functional form and parameter values in Tversky and Kahneman (1992).

	1	2	3	4	5	6	7	8	9	10	11
p				0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
f (BPs)		5	50	5	5	50	50	5	5	50	50
Model	B	F(5)	F(50)	L(2)	L(2)	L(2)	L(2)	L(1,2)	L(1,2)	L(1,2)	L(1,2)
$\mu = 1.05$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.09$	0.01	0	0.11	0	0	0.04	0.10	0	0	0.06	0.13
$\mu = 1.10$	1.09	0.73	1.25	0.72	0.73	1.16	1.23	0.73	0.74	1.18	1.26
$\mu = 1.11$	2.22	1.90	2.46	1.90	1.90	2.34	2.41	1.90	1.91	2.37	2.44
$\mu = 1.12$	3.66	3.14	3.73	3.14	3.14	3.59	3.66	3.14	3.15	3.62	3.70
$\mu = 1.13$	5.19	4.46	5.09	4.45	4.45	4.92	4.99	4.45	4.47	4.95	5.03
$\mu = 1.14$	6.97	5.86	6.54	5.85	5.85	6.33	6.41	5.86	5.87	6.37	6.45
$\mu = 1.15$	8.95	7.37	8.09	7.35	7.35	7.85	7.94	7.36	7.37	7.89	7.97
$\mu = 1.05$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0	0	0.40	0
$\mu = 1.07$	0	0	0	0	0	1.11	0	0	0	1.84	0.55
$\mu = 1.08$	0	0	0	0	0	2.47	0.73	0	0.46	3.32	2.46
$\mu = 1.09$	0	0	0	0.29	0.69	3.88	2.32	0.91	2.41	4.87	4.47
$\mu = 1.10$	1.11	0.69	1.30	1.70	2.33	5.37	3.99	2.43	4.46	6.51	6.57
$\mu = 1.11$	2.43	2.04	2.69	3.18	4.05	6.94	5.76	4.03	6.62	8.24	8.79
$\mu = 1.12$	4.08	3.49	4.18	4.75	5.88	8.60	7.63	5.73	8.90	10.07	11.13
$\mu = 1.13$	5.92	5.03	5.75	6.41	7.81	10.37	9.62	7.52	11.33	12.02	13.63
$\mu = 1.14$	8.01	6.66	7.44	8.19	9.88	12.26	11.74	9.44	13.91	14.09	16.30
$\mu = 1.15$	10.34	8.41	9.25	10.09	12.09	14.28	14.02	11.49	16.69	16.30	19.15

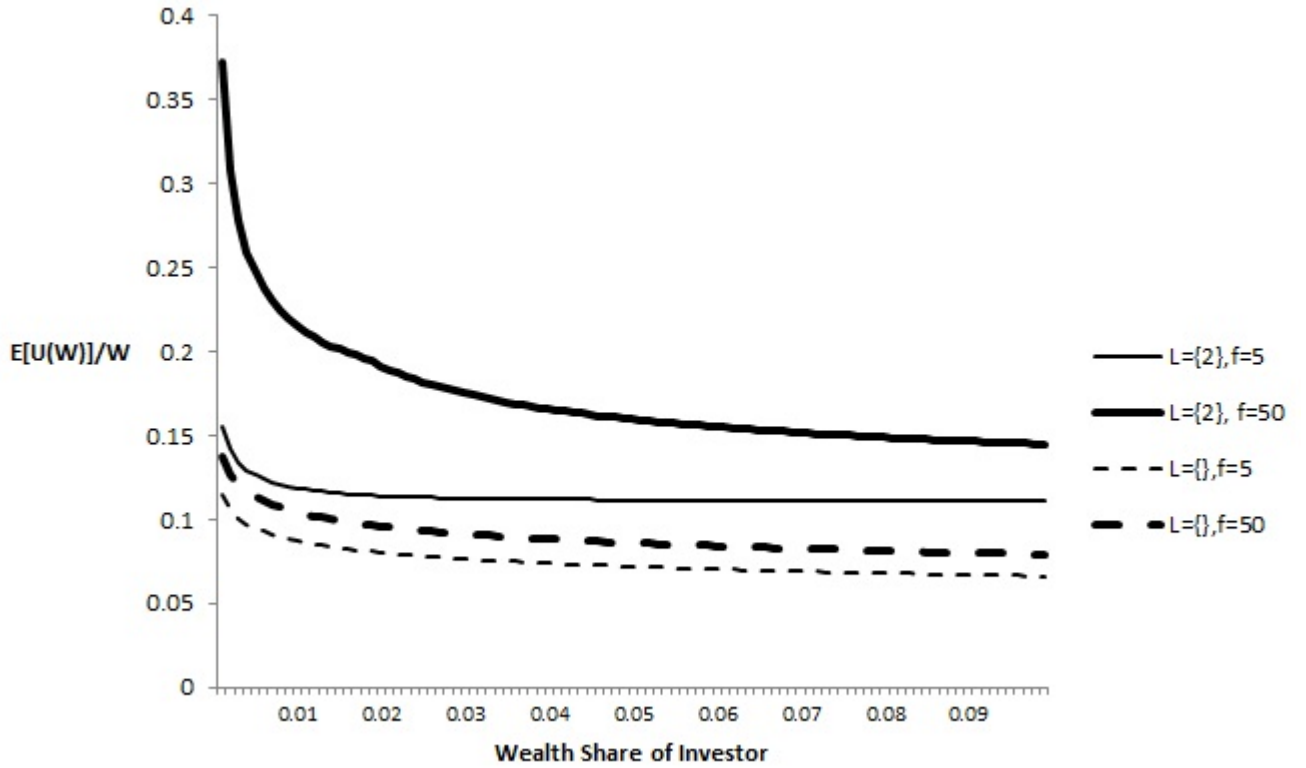
**Figure 1: Potential Gains from Applying Decision Weights to Lottery Payoffs** This figure shows the potential gains from calculating lottery payoffs with decision weights instead of objective probabilities. Tversky and Kahneman (1992) estimate the weighting function for nonnegative gambles as a two-part power function:  $w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$ ,  $\gamma = 0.61$ . Suppose a monopolist provides a lottery with a  $p$  probability of winning. The potential gains from offering the lottery are  $\max(w^+(p) - p, 0)$ . If the prospect theory investor overvalues the probability of winning the lottery, the monopolist offers the lottery at the actuarially fair price computed using decision weights and extracts all the gains. If the prospect theory investor undervalues the probability of winning the lottery, the monopolist will not offer the lottery.



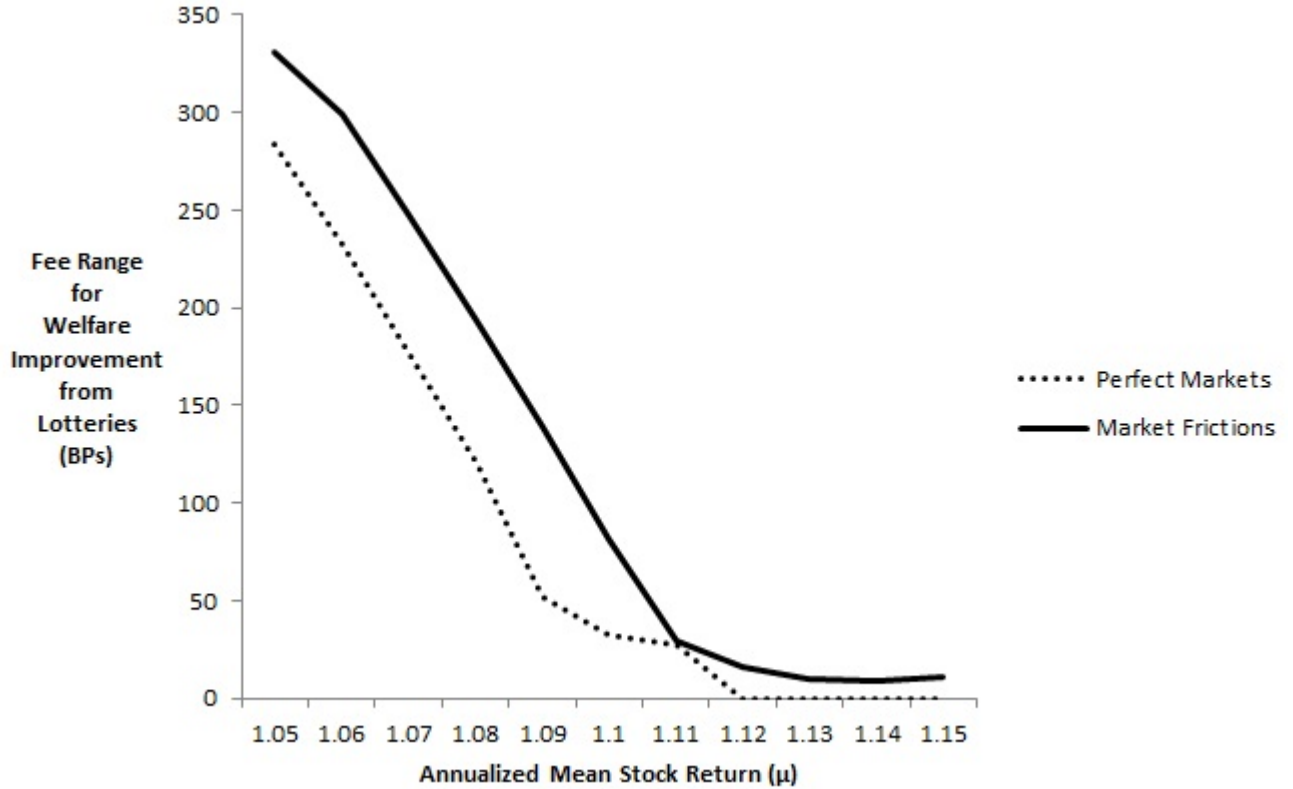
**Figure 2: Conditions for Welfare Gains** This figure shows conditions in which introducing stock loan fees or stock loan lotteries produces unconditional gains in investor welfare. An investor has unconditionally greater welfare if he has both higher expected utility and higher expected wealth. For all points above the dotted curve, the investor has unconditionally greater welfare by choosing the utility-maximizing allocation in the model with stock loan fees than by choosing the utility-maximizing allocation in the baseline model of realization utility. For all points above the solid line, the investor has unconditionally greater welfare from the utility-maximizing allocation in the model with stock loan lotteries than from the utility-maximizing allocation in the baseline model of realization utility. In the model with stock loan lotteries, there is a single lottery at  $t = 2$  with a probability  $p = 0.1$  of winning the lottery. The shaded region represents points in  $(\mu, f)$  space in which introducing stock loan lotteries leads to unconditional improvements in investor welfare.



**Figure 3: Regressive Features of Stock Loan Lotteries** This figure shows how introducing stock loan lotteries is regressive in that it provides disproportionate benefits to poor investors. In a stock loan marketplace where there is a single winner of a single lottery, the probability of winning the lottery is identical to the investor's wealth share. This figure shows how a standardized measure of utility,  $E[U(W_i)]/W_i$ , varies by the investor's wealth share, in four different two-period models of realization utility. The solid lines show standardized utility for models with a single stock loan lottery at  $t = 2$  and stock loan fees of five and 50 basis points. The dotted lines show standardized utility for models with no stock loan lotteries and stock loan fees of five and 50 basis points. For all models, the annualized expected gross return of the risky asset ( $\mu$ ), is 1.12.



**Figure 4: Stock Loan Lotteries with Transactions Costs and Leverage Constraints** This figure shows how the conditions for unconditional improvement in welfare change after introducing transactions costs and leverage constraints. An investor has unconditionally greater welfare if the utility-maximizing allocation provides strictly higher expected utility and strictly higher expected wealth. There are three two-period models of realization utility: the baseline Barberis and Xiong (2009) model, a model with fixed stock loan fees, and a model with a single lottery at  $t = 2$  with a probability  $p = 0.1$  of winning the lottery. For different values of  $\mu$ , the annualized expected gross return of the risky asset, I solve all three models in a perfect market as well as a market with frictions. The market frictions include 1.3% round-trip transactions costs and a maximum leverage of 2. For each value of  $\mu$ , in each environment, I calculate  $f_F$  and  $f_L$ , the minimum fee required to provide investors in the fee and lottery models unconditionally greater welfare than in the baseline model. For each  $\mu$ , the potential for welfare improvement by introducing stock loan lotteries is  $\max[f_F(\mu) - f_L(\mu), 0]$ . The solid line shows the potential for welfare improvement with market frictions, while the dotted line shows the potential for welfare improvement with perfect markets.





## Appendix: Proof of Proposition 1

The investor maximizes  $E_0v(x)$ , where  $v(x)$  has the functional form:

$$\begin{cases} v(x) = x^\alpha & x \geq 0, 0 < \alpha < 1 \\ v(x) = -\lambda(-x)^\alpha & x < 0, 0 < \alpha < 1, \lambda > 1 \end{cases}$$

Suppose the investor accepts some gamble ( $G$ ) with potential gains ( $g_1, g_2 \dots g_m$ ) and potential losses ( $l_1, l_2 \dots l_m$ ). This implies:

$$E_0v(x; G) = \sum_{i=1}^m p(g_i)v(g_i) + \sum_{j=1}^n p(l_j)v(l_j) > 0$$

Applying the functional form of the value function:

$$E_0v(x; G) = \sum_{i=1}^m p(g_i)(g_i)^\alpha - \lambda \sum_{j=1}^n p(l_j)(-l_j)^\alpha > 0$$

Consider a proportionately larger gamble ( $kG, k > 1$ ). This gamble has expected value:

$$E_0v(x; kG) = \sum_{i=1}^m p(g_i)v(kg_i) + \sum_{j=1}^n p(l_j)v(kl_j)$$

Applying the functional form of the value equation to the larger gamble:

$$E_0v(x; kG) = k^\alpha \sum_{i=1}^m p(g_i)(g_i)^\alpha - \lambda k^\alpha \sum_{j=1}^n p(l_j)(-l_j)^\alpha$$

$$E_0v(x; kG) = k^\alpha [\sum_{i=1}^m p(g_i)(g_i)^\alpha - \lambda \sum_{j=1}^n p(l_j)(-l_j)^\alpha] = k^\alpha E_0v(x; G)$$

Since  $k > 1, 0 < \alpha < 1$ , and  $E_0v(x; G) > 0$ ,

$$E_0v(x; Kg) > E_0v(x; g) > 0$$

So given that the investor accepts  $G$ , he always prefers  $kG$ . Therefore, an investor who chooses a positive risky asset allocation always chooses to exhaust the nonnegative wealth constraint.

QED