Student Research Symposium Posters

Apr 20th, 2:00 PM

Primordial Black Hole Atoms

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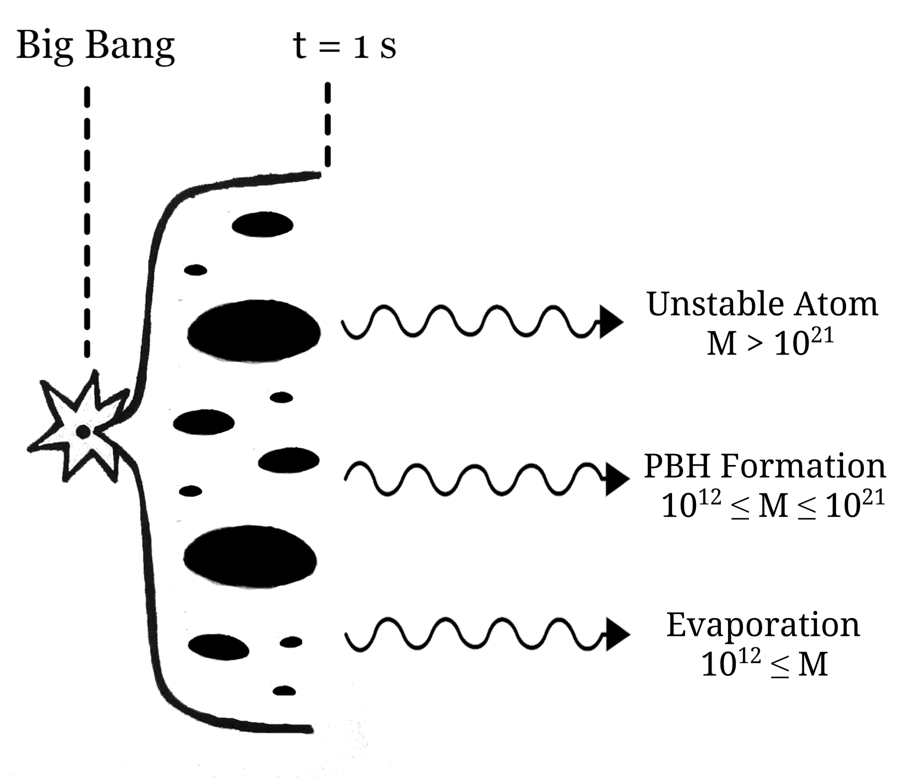
Abstract

Primordial black holes are thought to have been formed at the early stages of the universe in the presence of non-homogeneous density distributions of dark matter. We are working under the assumption that dark matter consists of elementary low mass particles, specifically, spin 1/2 fermions. We further assume that dark matter is electrically neutral, thus its main interaction is gravitational. We investigate dark matter spin 1/2 fermions in orbit around a black hole atom and consider mass ranges for which the quantum description is appropriate. Solutions to the Dirac equation are utilized to describe the radial mass distribution of primordial black hole atoms. Stable black holes atoms could be the seeds for galaxy formation.

Background & Theory

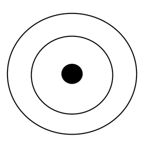
Cosmology

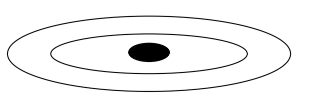


Classical Quantum Mechanics

Potential Equivalence

$$V = -\frac{kq}{r} \qquad U = -\frac{GM}{r}$$





Relativistic Quantum Mechanics

Dirac Equation

$$(i\gamma^{\mu}\partial_{\mu} - m + V)\,\psi \;=\; 0$$

Radial Dirac Wave Functions for a Central Potential

Involves Kummer Confluent Hypergeometric Functions

$$P_{n\kappa}(r) \;=\; \sqrt{1+\epsilon_{n\kappa}/mc^2}\; N_{n\kappa} e^{-x/2} x^{\gamma}\,[(N-\kappa)F(-n+k, 2\gamma+1, x) - (n-k)F(-n+k+1, 2\gamma+1, x)]\,,$$

$$Q_{n\kappa}(r) \;=\; \sqrt{1-\epsilon_{n\kappa}/mc^2}\; N_{n\kappa} e^{-x/2} x^{\gamma}\,[(N-\kappa)F(-n+k, 2\gamma+1, x) + (n-k)F(-n+k+1, 2\gamma+1, x)]\,.$$

Normalized with Gamma Functions

$$N_{n\kappa} = \frac{1}{N\,\Gamma(2\gamma+1)}\sqrt{\frac{Z\,\Gamma(2\gamma+1+n-k)}{2\,(n-k)!\,(N-\kappa)}}$$

Energy Levels

$$E_{n\kappa} = \frac{mc^2}{\sqrt{1+\left(\frac{b}{n-k+\gamma}\right)}}$$

Approximations & Assumptions

Spherical Symmetry

$$\epsilon_{n\kappa} \approx mc^2 \implies Q = 0$$

$$\kappa \;\equiv\; [-n, 0) \;\cup\; (0, n-1]$$

$$k \approx \gamma$$

$$x = 2\lambda r$$

$$R_s = \frac{2GM}{c^2}$$

$$b > 1 \implies \text{unstable atom}$$

$$b = \frac{GmM}{\hbar c}$$

$$b < \frac{1}{100}$$

$$\lambda = \sqrt{m^2c^4 - \left(\frac{E_{n\kappa}}{\hbar c}\right)} \approx \frac{mc^2}{\hbar c}\left(\frac{b}{n}\right)$$

Results

- Because of Mathematica's limitations, only graphs up to n = 32 could be produced accurately.
- The total number of particles if all the states up to η are occupied is given by:

$$N(\eta) = \sum_{n=1}^{\eta} 2n^2 = \frac{1}{3}\eta(1+\eta)(1+2\eta)$$

- For the case that all levels up to n = 32 are occupied, there are 22,880 particles.

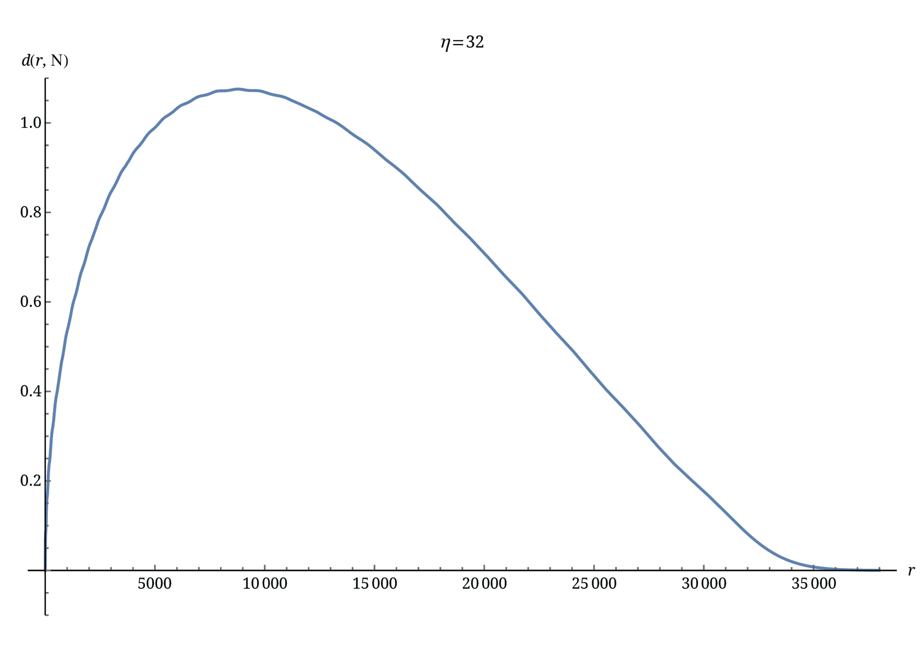
- The radial probability function for the mass distribution of the fermions is defined as:

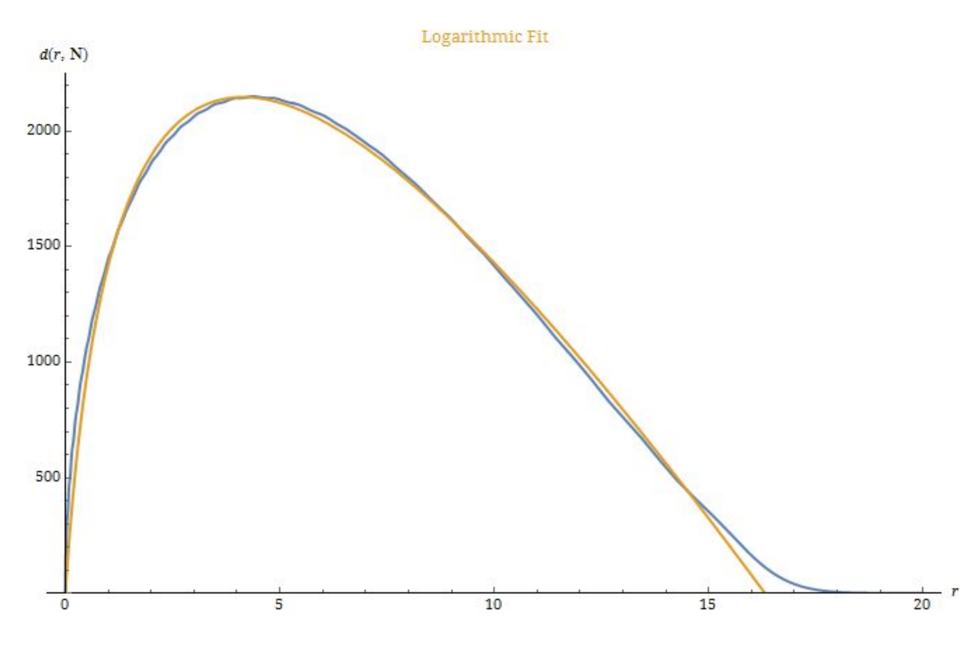
$$P(r) \;=\; \sqrt{2}N_{n\kappa}e^{2\lambda r}x^{k}[(n-\kappa)F(-n+k, 2k+1, x) - (n-\kappa)F(-n+k+1, 2k+1, x)]$$

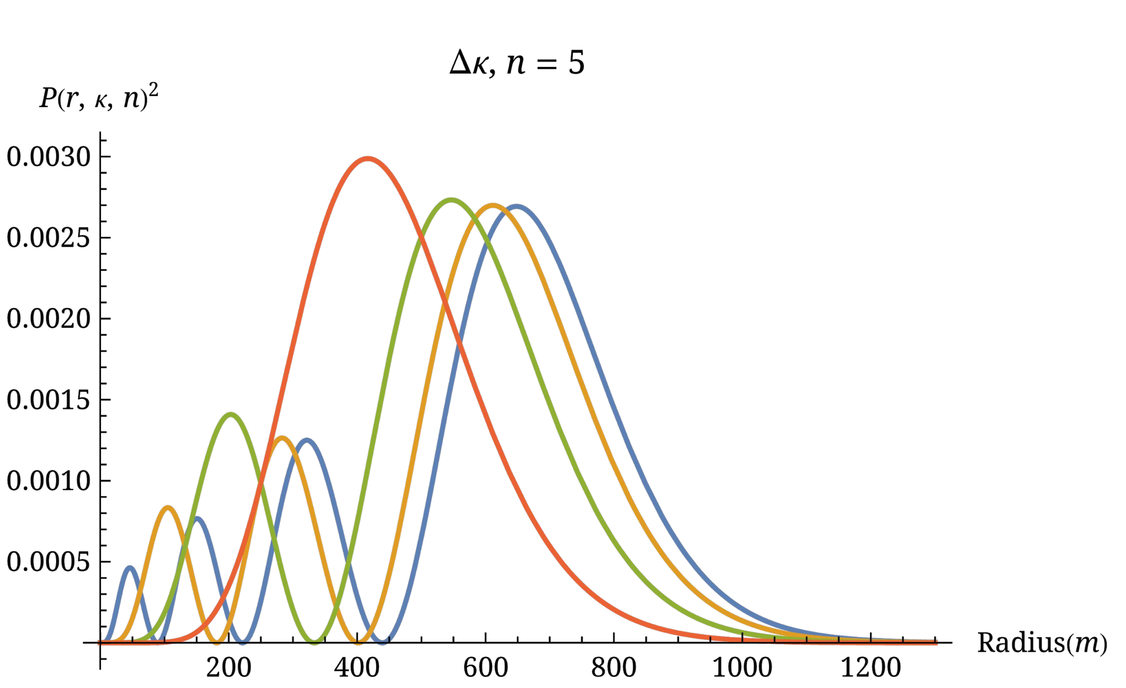
- The summation of the probability functions for multiple n values is expressed as:

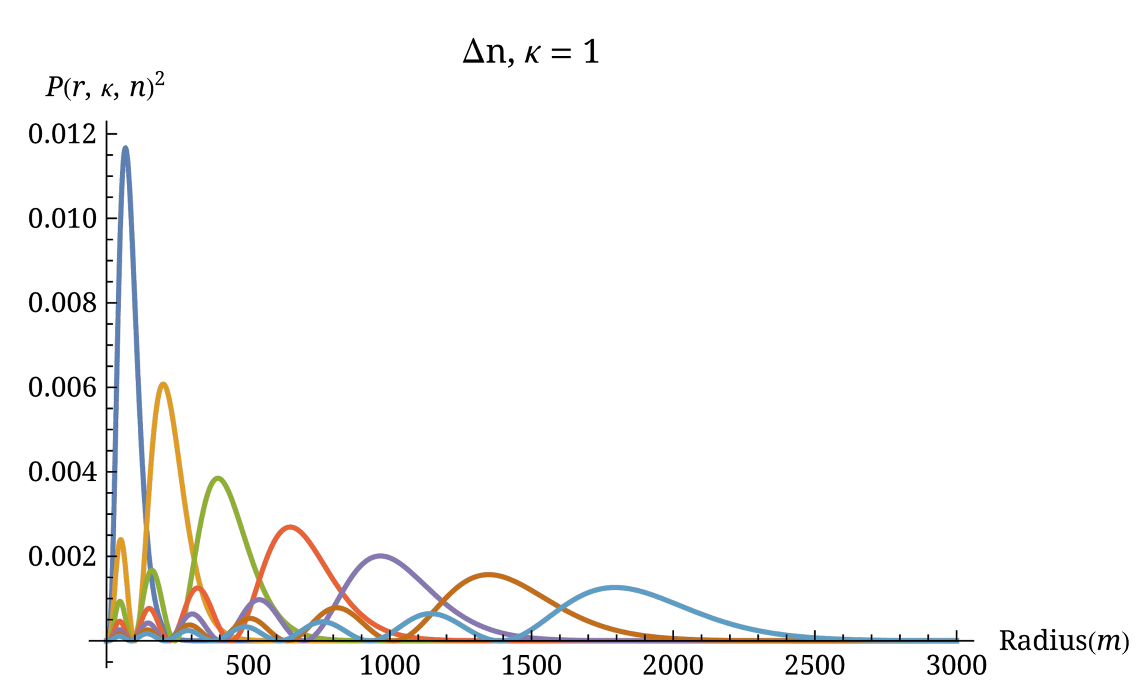
$$d(r, N) = \sum_{n=1}^{\eta}[2P^2_{n,-1} + \sum_{\kappa=2}^{\eta}(2\kappa-1)P^2_{n,-\kappa} + \sum_{\kappa=1}^{n-1}(2\kappa+1)P^2_{n,\kappa}]$$

m \ M	10^{12} ($R_s = 1.5 \times 10^{-15}$)	10^{15} ($R_s = 1.5 \times 10^{-12}$)	10^{18} ($R_s = 1.5 \times 10^{-9}$)	10^{21} ($R_s = 1.5 \times 10^{-6}$)
~ 10^{-38}	$b = 2.1 \times 10^{-11}$, $R = 3.5 \times 10^{9}$	$b = 2.1 \times 10^{-8}$, $R = 3.5 \times 10^{6}$	$b = 2.1 \times 10^{-5}$, $R = 3.5 \times 10^{3}$	$b = 2.1 \times 10^{-2}$, $R = 3.5$
~ 10^{-37}	$b = 1.27 \times 10^{-9}$, $R = 10^{6}$	$b = 1.27 \times 10^{-6}$, $R = 10^{3}$	$b = 1.27 \times 10^{-3}$, $R = 1$	~~$b = 1.27$~~
~ 10^{-30}	$b = 2.1 \times 10^{-3}$, $R = 3.5 \times 10^{-7}$	~~$b = 2.1$~~	~~$b = 2.1 \times 10^{-11}$~~	~~$b = 2.1 \times 10^{-11}$~~
~ 10^{-27}	~~$b = 3.52$~~	~~$b = 3.52 \times 10^{3}$~~	~~$b = 3.52 \times 10^{6}$~~	~~$b = 3.52 \times 10^{9}$~~
~ 10^{-24}	~~$b = 2110$~~	~~$b = 2.1 \times 10^{6}$~~	~~$b = 2.1 \times 10^{9}$~~	~~$b = 2.1 \times 10^{12}$~~









Future Work

In the future we will attempt to find an exact equation to express the probability distribution at any range of n's. Our focus will turn to adding potential terms to account for the accumulated mass of the fermions at high n's. Numerical calculations will be necessary to better represent the data at values of b > 1.

Chapter 3 Accurate Relativistic Calculations Including QED.... WR Johnson, KT Cheng, MH Chen