Integrating the curriculum using a small-scale hybrid power train

Mariaeugenia Salas Acosta

Follow this and additional works at: https://rdw.rowan.edu/etd

Part of the Mechanical Engineering Commons

Let us know how access to this document benefits you - share your thoughts on our feedback form.

Recommended Citation
Salas Acosta, Mariaeugenia, "Integrating the curriculum using a small-scale hybrid power train" (2015). Theses and Dissertations. 274.
https://rdw.rowan.edu/etd/274

This Thesis is brought to you for free and open access by Rowan Digital Works. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of Rowan Digital Works. For more information, please contact LibraryTheses@rowan.edu.
INTEGRATING THE CURRICULUM USING A SMALL-SCALE HYBRID POWER TRAIN

by

Mariaeugenia Salas Acosta

A Thesis

Submitted to the
Department of Mechanical Engineering
College of Engineering
In partial fulfillment of the requirement
For the degree of
Master of Science in Engineering
at
Rowan University
Jun 2, 2014

Thesis Chair: Dr. Eric Constans, Ph.D.
Dedication

To my Family especially to my parents who with love and effort have given me the opportunity to have a great education; as well as their support at all times

To Carlos who has continuously encourage me during this process with love and patience
Acknowledgments

First of all I am grateful to God for establishing me to complete this Master’s Degree. I would like to express my deepest appreciation to my advisor Dr. Eric Constans for giving me the opportunity of being part of this project. He continuously and convincingly conveyed a spirit of excitement regarding to research and scholarship. Without his guidance, patience and persistence this thesis would not have been possible. I would also want to express my gratitude to: Dr. Hong Zhang, Dr. Thomas Merrill, Dr. Krishan Bhatia and Dr. Jennifer Kadlowec, for supporting me during this process. I was very lucky to meet this team of wonderful and dedicated Professors. I would also want to thank Charles Linderman (Chuck) and Karl Dyer for giving me a hand in the shop whenever I needed it. Also, to Barbara Wynn who was very diligent with all my paperwork.

I can say that the mechanical engineering department at Rowan University left a mark in my life and I will never forget this enriching experience.

Thank you!
Abstract

Mariaeugenia Salas Acosta
NTEGRATING THE CURRICULUM USING A SMALL-SCALE HYBRID POWER TRAIN
2013/2014
Eric Constans, Ph.D.
Master of Science in Engineering

The project, a bench-scale hybrid electric powertrain system, is designed, analyzed and fabricated by students in six modules, starting in their sophomore year and culminating in their final semester as seniors. This complex project has been selected in order to integrate the core mechanical engineering courses: Mechanical Design, Thermodynamics, System Dynamics and Control, and Fluid Mechanics. A bench-scale hybrid-electric vehicle powertrain has sufficient complexity to involve all Mechanical Engineering disciplines and the simplicity to be built by students over the course of five semesters. The work is designed to test two hypotheses:

• A long-term design project that integrates knowledge from multiple courses strengthens student knowledge retention.

• A large-scale design project requiring tools from many courses improves student problem-solving and design skills.

By integrating five semesters of the mechanical engineering curriculum into a cohesive whole, this project has the potential to transform the way undergraduate education is delivered. Before and after testing is being conducted to assess: a) Change in retention
Abstract (Continued)

Mariaeugenia Salas Acosta
INTEGRATING THE CURRICULUM USING A SMALL-SCALE HYBRID POWER TRAIN
2013/2014
Eric Constans, Ph.D.
Master of Science in Engineering

between courses and b) Change in student problem-solving and design skills.

Students at Rowan University have built all of the “hardware” for the HPT (air engine, planetary gearset, tachometer, etc.) in earlier semesters. The control system is the “capstone” for the five-semester design project, which has been described in an earlier publication [1].

This thesis describes the development of the “faculty prototype” of the control system, and gives preliminary results of implementing the control system design project in the classroom. Also, it includes the mathematical behavior of each of the components for better understanding of the reader.
## Table of Contents

Abstract

List of Figures

List of Tables

Chapter 1. Project Background

Section 1.01 Project Background

Section 1.02 Main Purpose of this Thesis

Chapter 2: Literature

2.01 Toyota Hybrid System THS II

2.02 Modeling and Analysis of the Toyota Hybrid System

Chapter 3: Hybrid System

3.01 Toyota Hybrid System (THS)

3.02 Hybrid Power Train (HPT) System

Chapter 4: Planetary Gears

4.01 Planetary Gears Theory

4.02 Simplifying the Model

4.03 Some Additional Notes

Chapter 5: Electric Motor

5.01 Electric Motor Theory

5.02 Calculating Electric Motor Constants
Table of Contents (Continued)

5.03 Including the Electric Motor in the HPT System 41

Chapter 6: Air Engine 43

6.01 Air Engine Theory 43

6.02 Experimental Appr.to determine the Air Engine Parameters 46

6.03 Determining the Air Engine Optimal Speed 49

6.04 Including the Air Engine to the Hybrid Power Train System 53

Chapter 7: Load Torque from the Generator 55

7.01 Generator Theory 55

7.02 Experimental Approach to find the Back-emf constant (Kb) 59

7.03 Including the experimental data 61

Chapter 8. Load Box 62

8.01 Load Box Control Module 62

8.02 Adding Power to the system 64

8.03 Simulation of the Load Box 65

Chapter 9. Battery Box 66

9.01 Open-Circuit Voltage vs. State of Charge 66

9.02 State of Charge using Coulomb-Counting 69

9.03 Generator and Battery Box Control Module: Hardware 70

Chapter 10: Hardware Design for the control of the HPTS 73
Table of Contents (Continued)

10.01 Sensing the speed of the motors 73
10.02 Air Flow Control Module 73
10.03 Current Control Module 78
Chapter 11: Control Module of the Hybrid Power Train System 80
11.01 Control Module of the Hybrid Electric Power-Train 80
11.02 Decision Making Algorithm 82
11.03 Set Value: Air and Electric Motors 86
11.04 PID controller 87
11.05 Note PID controller 92
11.06 First Estimation of the Require Velocity 92
11.07 Battery 94
11.08 Load Box 96
Chapter 12: Implementing the HPT in a Classroom Setting 97
12.01 Providing the students with a Station 97
12.02 Successfully Implemented Modules 98
Chapter 13: Conclusions 105
List of Reference 107
Appendix A: Air Engine Efficiency 110
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3.1</td>
<td>THS Configuration</td>
<td>12</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>System Operation</td>
<td>13</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Planetary Gear Prius</td>
<td>14</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>HPT Overhead View</td>
<td>15</td>
</tr>
<tr>
<td>Figure 3.5</td>
<td>HPT System Configuration</td>
<td>16</td>
</tr>
<tr>
<td>Figure 3.6</td>
<td>Power and Electrical Paths HPT</td>
<td>17</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>Planetary Gear Train</td>
<td>19</td>
</tr>
<tr>
<td>Figure 4.2</td>
<td>Schematic of a Differential Gear</td>
<td>21</td>
</tr>
<tr>
<td>Figure 4.3</td>
<td>Planetary Gear used for the Project</td>
<td>22</td>
</tr>
<tr>
<td>Figure 4.4</td>
<td>Free-body Diagram Sun Gears</td>
<td>23</td>
</tr>
<tr>
<td>Figure 4.5</td>
<td>Dimension used for the Model</td>
<td>24</td>
</tr>
<tr>
<td>Figure 4.6</td>
<td>Free-body Diagram Planetary Gears</td>
<td>25</td>
</tr>
<tr>
<td>Figure 4.7</td>
<td>Free-body Diagram Planetary Carrier</td>
<td>26</td>
</tr>
<tr>
<td>Figure 5.1</td>
<td>Theoretical DC Motor</td>
<td>29</td>
</tr>
<tr>
<td>Figure 5.2</td>
<td>Electric Motor used for the small-scale HPT</td>
<td>31</td>
</tr>
<tr>
<td>Figure 5.3</td>
<td>Pulley-Mass System</td>
<td>33</td>
</tr>
</tbody>
</table>
**List of Figures (Continued)**

<table>
<thead>
<tr>
<th>Figure Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 5.4 Electric Motor Behavior during Experiment</td>
<td>34</td>
</tr>
<tr>
<td>Figure 5.5 Verification of the EM's Mechanical Constants</td>
<td>35</td>
</tr>
<tr>
<td>Figure 5.6 Step Response of the EM with 24V Applied</td>
<td>38</td>
</tr>
<tr>
<td>Figure 5.7 Experimental vs. Theoretical Data</td>
<td>39</td>
</tr>
<tr>
<td>Figure 6.1 Diagram of the Air Flow Control Module</td>
<td>43</td>
</tr>
<tr>
<td>Figure 6.2 Theoretical Air Engine</td>
<td>44</td>
</tr>
<tr>
<td>Figure 6.3 Air engine used for the Power Train System</td>
<td>46</td>
</tr>
<tr>
<td>Figure 6.4 Air Engine Behavior during Experiment</td>
<td>47</td>
</tr>
<tr>
<td>Figure 6.5 Experimental Vs Theoretical Data</td>
<td>48</td>
</tr>
<tr>
<td>Figure 6.6 Angular Velocity Vs Torque. Velocity n=35</td>
<td>49</td>
</tr>
<tr>
<td>Figure 6.7 Air Engine’s Efficiency</td>
<td>52</td>
</tr>
<tr>
<td>Figure 7.1 Generator theoretical model</td>
<td>55</td>
</tr>
<tr>
<td>Figure 7.2 Generator used for the small-scale HPT</td>
<td>56</td>
</tr>
<tr>
<td>Figure 7.3 Electric Motor dimension in (mm)</td>
<td>56</td>
</tr>
<tr>
<td>Figure 8.1 Load Box Circuit</td>
<td>63</td>
</tr>
<tr>
<td>Figure 9.1 Battery Diagram</td>
<td>66</td>
</tr>
</tbody>
</table>
# List of Figures (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 9.2</td>
<td>Current Sensor</td>
<td>70</td>
</tr>
<tr>
<td>Figure 9.3</td>
<td>Current Sensor Connections</td>
<td>70</td>
</tr>
<tr>
<td>Figure 9.4</td>
<td>Temperature sensor (TMP36)</td>
<td>71</td>
</tr>
<tr>
<td>Figure 9.5</td>
<td>Voltage Divider</td>
<td>72</td>
</tr>
<tr>
<td>Figure 10.1</td>
<td>Hall Effect Tachometer</td>
<td>73</td>
</tr>
<tr>
<td>Figure 10.2</td>
<td>Diagram of the Air Flow Control System</td>
<td>74</td>
</tr>
<tr>
<td>Figure 10.3</td>
<td>Push-to-Connect Male Pipe Adapter</td>
<td>76</td>
</tr>
<tr>
<td>Figure 11.1</td>
<td>Microcontroller Arduino Mega 2560</td>
<td>80</td>
</tr>
<tr>
<td>Figure 11.2</td>
<td>Control Schematic</td>
<td>81</td>
</tr>
<tr>
<td>Figure 11.3</td>
<td>Hybrid Power Train Algorithm</td>
<td>85</td>
</tr>
<tr>
<td>Figure 11.4</td>
<td>Air Engine Algorithm</td>
<td>89</td>
</tr>
<tr>
<td>Figure 11.5</td>
<td>Electric Motor Algorithm</td>
<td>91</td>
</tr>
<tr>
<td>Figure 11.6</td>
<td>Inverse of the electric motor behavior</td>
<td>92</td>
</tr>
<tr>
<td>Figure 11.7</td>
<td>Air Engine Behavior Step by Step</td>
<td>93</td>
</tr>
<tr>
<td>Figure 11.8</td>
<td>Inverse Air Engine Graph</td>
<td>94</td>
</tr>
<tr>
<td>Figure 12.1</td>
<td>Planetary Gearset used for the HPT prototype</td>
<td>100</td>
</tr>
</tbody>
</table>
List of Figures (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 12.2 Other planetary gearsets</td>
<td>101</td>
</tr>
<tr>
<td>Figure 12.3 Student-designed and built planetary gearset</td>
<td>102</td>
</tr>
<tr>
<td>Figure 12.4 Solenoid valves and orifice block</td>
<td>103</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.1 Design/Build Project Timeline</td>
<td>4</td>
</tr>
<tr>
<td>Table 1.2 Project evaluation time-line</td>
<td>4</td>
</tr>
<tr>
<td>Table 3.1 Differences between THS and HPT</td>
<td>18</td>
</tr>
<tr>
<td>Table 4.1 Rotational shaft linkages of the planetary gear</td>
<td>21</td>
</tr>
<tr>
<td>Table 4.2 Components for the Dynamic Analysis</td>
<td>23</td>
</tr>
<tr>
<td>Table 5.1 Electric Motor Constant Parameters</td>
<td>40</td>
</tr>
<tr>
<td>Table 6.1 Air Engine Constant Parameters</td>
<td>47</td>
</tr>
<tr>
<td>Table 6.2 Units of the Variables</td>
<td>52</td>
</tr>
<tr>
<td>Table 6.3 Air engine Values</td>
<td>54</td>
</tr>
<tr>
<td>Table 7.1 Data obtained to calculate the Generator’s Kb</td>
<td>60</td>
</tr>
<tr>
<td>Table 7.2 Values obtained for the system</td>
<td>61</td>
</tr>
<tr>
<td>Table 8.1 FET’s Control Signals</td>
<td>64</td>
</tr>
<tr>
<td>Table 10.1 Binary Pattern</td>
<td>75</td>
</tr>
<tr>
<td>Table 10.2 Orifice Sizes Flow Control Module #1</td>
<td>77</td>
</tr>
<tr>
<td>Table 10.3 Orifice Sizes Flow Control Module #2</td>
<td>77</td>
</tr>
<tr>
<td>Table 11.1 Scenarios</td>
<td>84</td>
</tr>
</tbody>
</table>
List of Tables (Continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 11.2 Signals Arduino</td>
<td>96</td>
</tr>
<tr>
<td>Table 12.1 Design/Build Project Timeline</td>
<td>99</td>
</tr>
<tr>
<td>Table 13.1 Scenarios</td>
<td>105</td>
</tr>
<tr>
<td>Table 13.2 Responses</td>
<td>106</td>
</tr>
</tbody>
</table>
Chapter 1

Project Background

Section 1.01  Project Background

Professors are trying to find the way to improve the student’s ability to retain critical engineering concepts. Despite their efforts it is common for students to forget important information learned in previous semesters. A description of this phenomenon is given by Avitabile. [2]

*The unfortunate part is that as soon as the test is over or the course is completed, the students often just forget the material since they have no reason to retain the compartmentalized, modularized material.*

Disciplines which in practice go hand by hand are being separated in the mechanical engineering curriculum. Subjects such as thermodynamics and mechanical design are isolated in the curriculum when in practice are integrated. Thermal and mechanical systems must function cohesively in real mechanical systems (e.g. an air conditioner).

Many sources have made the case for reforming engineering education to reflect modern trends. Most notably, a recent National Academy of Engineering (NAE) report found that [3]

*Engineering education must avoid the cliché of teaching more and more about less and less, until it teaches everything about nothing. Addressing this problem may involve reconsideration of the basic structure of engineering departments and the infrastructure for evaluating the performance of professors as much as it does selecting the coursework students should be taught.*

Teaching sustainable design and ecologically-friendly practices is another important subject that the report addresses.
With this in mind, the Mechanical Engineering department at Rowan University has implemented a novel, potentially transformative approach to integrating coursework through five semesters of the core mechanical engineering curriculum. The proposed work will test two hypotheses:

1. A long-term design project that integrates knowledge from multiple courses strengthens student knowledge retention.
2. A large-scale design project requiring tools from many courses improves student problem-solving and design skills.

By integrating five semesters of the mechanical engineering curriculum into a cohesive whole, this project has the potential to transform the way undergraduate education is delivered. Before and after testing will be conducted to assess: a) Change in retention between courses and b) Change in student problem-solving and design skills.

A challenging project was developed to integrate the core mechanical engineering courses which include: Machine Design, Mechanical Design, Thermal-Fluid Sciences and System Dynamics. A Hybrid Electric Vehicle Powertrain has been found to have the required complexity to involve all the disciplines but at the same time the simplicity to be built by students over the course of five semesters. Another reason to select this project is that hybrid technology is considered to be at the cutting-edge of the automobile industry.

At the present time, most of the automobile manufactures have included ecofriendly cars to their portfolio. There are two main reasons. The first one is to make cars less dependent on petroleum. Recent increases in energy costs, especially for fossil fuels, have made highly fuel-efficient vehicles very desirable. The second one is to protect the environment. Since problems associated with greenhouse gas emissions,
especially global warming, demand innovative technologies, one promising solution to this crisis is the use of vehicles with hybrid gasoline/electric powertrains.

Toyota is well known to be at the edge of developing clean and safe products. The Toyota Prius is one of the most famous Hybrid Vehicles (HV) that is in the market. [4] It must be noted that during 2013, hybrid electric vehicles had a market share of 3.19% in U.S. [5]

The Toyota Prius was the first mass-produced hybrid passenger vehicle in the world. It was introduced in Japan in 1997 and in 2000 the overseas marketing began. The Prius has cumulative worldwide sales exceeding 110,000 units [4]. Toyota has positioned hybrid powertrains as its key technology.

The Hybrid vehicles are the fastest growing segment of the light-duty vehicle market [5]. Since it is a growing segment, US need to capitalize on this growth opportunity. To achieve this, US engineering students must become more proficient in the kind of multidisciplinary design that created hybrid-electric vehicles.

It must be mentioned that this work addresses two of the main concerns of the NAE report [3]: modernization of the engineering curriculum through multidisciplinary integration and stressing “green engineering” at the design level.

The hybrid-electric powertrain project is divided into six discrete modules, each of which can be completed in a few regularly-scheduled laboratory periods. Table 1.1 depicts the modules, the course, and the semester when it is to be completed. Table 1.2 shows the assessment schedule. The cohort of students preceding this project was used as a control group, and three successive cohorts of students will be used as test subjects.
The first cohort of students began designing/building the first module (the Tachometer) as sophomores in the Spring of 2012. This cohort is now in its senior year, and will finish the complete system in the Spring of 2014. The first four modules have been tested on students, and modules V and VI will be completed in the Spring of 2014.

<table>
<thead>
<tr>
<th>Table 1.1: Design/Build Project Timeline</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year</strong></td>
</tr>
<tr>
<td>1 (2011-2012)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2 (2012-2013)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3 (2013-2014)</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The first cohort of students began designing/building the first module (the Tachometer) as sophomores in the Spring of 2012. This cohort is now in its senior year, and will finish the complete system in the Spring of 2014. The first four modules have been tested on students, and modules V and VI will be completed in the Spring of 2014.

<table>
<thead>
<tr>
<th>Table 1.2: Project evaluation time-line</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year/Sem</strong></td>
</tr>
<tr>
<td>1 Spring</td>
</tr>
<tr>
<td>2 Fall</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3 Fall</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Assessment

As the assessment effort is ongoing, only a brief outline will be presented here. The assessment strategy is intended to determine the validity of the two hypotheses; thus the change in concept retention between semesters and the increase (or decrease) in design and problem-solving skills must be measured.

a) **Measuring the change in concept retention.** At the end of each course, the students are given a concept inventory test. These assessment tools have been externally validated [6] [7] and measure conceptual understanding for each of the respective content areas. The class in the year ahead of the test cohort was used as the control group, and was given the same concept inventory tests during the year preceding the formal study. It is expected that the first test cohort will show higher knowledge retention, based on average concept test score, than the control group. Also, for each of the three cohorts in this study, the average test scores to determine level of retention longitudinally will be compared.

b) **Measuring the change in students’ “Design Toolkit”.** McKenna [8] and Atman, et al. used design scenarios [9] and coding of student responses for investigating adaptive expertise and transfer of design process knowledge. In a similar vein, Harvey and Kadlowec [10] used writing analysis to examine students’ understanding of the design process. A similar approach has been adopted for this work, but the focus is on how the students use disciplinary knowledge rather than design process knowledge in the development of their solutions, thus showing whether or not they are able to make connections between subjects and retain knowledge between courses.
The change in design skill is being evaluated through the use of “rapid-design” projects – very small, open ended design scenarios in which students will be faced with the types of tasks they may encounter in practice. These scenarios are being given in all five courses to determine how likely students are to 1) draw upon concepts and skills learned in the particular course in which the design scenario was given and 2) draw upon concepts and skills that were not taught in that particular course, but in previous courses. In the investigators’ experience, the students should be better at doing the former than the latter, but the integrated project may improve the students’ ability in the latter compared to the control group.

Responses to the scenarios will be coded according to a rubric of specific learning outcomes. In the case of this investigation, the learning outcomes are core concepts in the courses previously taken in the Mechanical Engineering curriculum and that are part of the project. A five point Lickert scale is used to code evidence and appropriate use of disciplinary concepts from 1- unsatisfactory to 5- highly skillful. A two-way ANOVA will be used to compare the results before and after each phase of the design experience. The pre and post testing will allow for each student to serve as his/her own control. The ANOVA will allow us to test our hypothesis to determine if concepts learned in the immediately completed course improved student learning of those course concepts, if students retained concepts from a course prior to the immediately completed course and the interaction of these effects.

Section 1.02 Main Purpose of this Thesis

The two proposed hypothesis in the previous section will be tested after the students are done with the project. In order to achieve this goal the staff has to develop a
prototype for educational purposes. That is why the main objective of this thesis is to build a prototype of the *Hybrid Electric Power Train* that will serve as a model to students during their journey.

This thesis describes how each of the modules mentioned in Table 1.1 is integrated to the system. More specifically it describes the analytical behavior of each of the components of the hybrid power train (HPT) and its control algorithm. That means that the mathematical behavior of each component has been determined in order to understand the system thoroughly. Finally, it gives recommendations that will be useful for the students during its development.

In conclusion, this thesis will serve as a guideline for both professors and students during the execution of the hybrid powertrain project.
Chapter 2

Literature

Project based learning (PBL) can be defined as an instructional method in which the students have to learn the necessary concepts and develop a diverse set of skills in order to solve a given problem [11]. PBL provides the opportunity to expose students to real-life complex projects. Some of the advantages listed by numerous authors Cordray, et al. [12], Roselli and Brophy [13], Bransford, et al. [14], Jiusto and DiBiasio [15], Vanasupa, et al. [16] and Benjamin and Keenan [17] are that PBL has been found to:

- Significantly enhance the students’ understanding of difficult concepts
- Increase the engagement of the students with the course.
- Prepare students for lifelong and self-directed learning
- Increase the development of communication and teamwork skills.
- Promote creativity and help students develop problem solving skills.

PBL can be used as an opportunity to introduce projects that may integrate core courses that are separate in the engineering curriculum but are linked in real life [18]. This allows the students to integrate the knowledge from the different disciplines and helps them to understand complex theoretical concepts.

The method proposed has the potential of increasing involvement of underrepresented groups in mechanical engineering. According to Bush-Vishniak and Jarosz [19], the engineering environment could be more welcoming if first, students are
exposed to real-life concepts; second, courses are related; and third, the team oriented activities are increased.

Favorable reports of small-scale integration courses during the first years of the engineering curriculum have been proposed by researches Froyd and Ohland [20], DeBartolo and Robinson [21] and Marchese, et al [22]

The PBL method has been successfully implemented at Rowan University using small-scale integration courses. The engineering program is recognized for its multidisciplinary design-build projects. The Engineering Clinics [23] are a hallmark of the curriculum. The faculty has the experience in coordinating multidisciplinary projects. In addition, the Mechanical Engineering department has successfully integrated traditionally separated courses in the curriculum by implementing collaborative projects between courses. For instance, the Mechanical Design and Thermodynamics were integrated by implementing a design/built/test project: a small air compressor [24] and an air-powered engine [25]. Other project such as an electronic motor speed controller has also been successfully implemented [26].

The Mechanical Engineering department at Rowan’s University has begun to implement a long-term PBL in their curriculum. The faculty at Rowan University wants to test whether conducting a long-term (5 semester) project that comprises knowledge from different disciplines will improve students learning, design skills and knowledge retention from previous courses. A long-term design and build project that integrates six core courses is not documented in the literature; thus, it appears to be a novel approach.
The project selected is a small-scale hybrid power train. It has the required complexity to include various courses and the desired simplicity to last for five semesters. The students seem to be fascinated by this cutting edge technology. The Faculty at Rowan University has the experience and confidence to undertake this challenging and potentially transformative project.

Section 2.01  Toyota Hybrid System THS II

Toyota has characterized itself for developing and providing environmental friendly products. The Toyota Hybrid System was introduced in Japan in 1997 as an ecofriendly power train. The THS was installed in the passenger vehicle Prius which was introduced that same year. It was the first mass-produced hybrid vehicle in the market. [4].

The powertrain prototype built in this project is similar to the Toyota Hybrid System. It is a small-scale prototype for practical and educational purposes.

Section 2.02  Modeling and Analysis of the Toyota Hybrid System

A dynamic model and analysis of the Toyota Hybrid System has been described by Jimming Liu, et al. [27] It was a model using the software Matlab/Simulink, including a rule-based control strategy described by Hermance [28], [29] and [4].

A similar approach has been developed for the Rowan’s Prototype. A ruled-based control was also implemented. System parameters were found experimentally in order to simulate the powertrain using the software Matlab/Simulink.
Chapter 3

Hybrid System

This chapter provides an explanation of hybrid powertrain technology. Specifically, it describes the operation of the Toyota Hybrid System (THS). Furthermore, it gives an overview of the Hybrid Power Train (HPT) prototype built at Rowan University and its main differences between the HPT and the THS.

Section 3.01 Toyota Hybrid System (THS)

Toyota is widely known for developing cutting-edge safe and environmentally friendly products. They have developed the Toyota Hybrid System (THS) which combines a gasoline engine and an electric motor, with the advantage of not requiring external charging. According to the Toyota’s Publication THS II [4] the system achieves nearly twice the fuel efficiency of conventional gasoline engines. This system was included in the Toyota Prius. This was the first hybrid vehicle (HV) in the world that was mass produced. The Prius has exceeded the 110,000 sales worldwide since its release in Japan in 1997. [4]

The THS uses a gasoline engine, electric motor, and electric generator to achieve better fuel efficiency which results in reducing exhaust emissions of carbon dioxide (CO₂). This system has two motive power sources which engage depending on the driving conditions. The gasoline engine is the main power to move the wheels. The AC electric motor is power by a nickel-metal hybrid battery and an AC electric generator. This motor also provides power to drive the wheels of the car when the gasoline engine is not working at its optimal conditions. [30]
The THS system divides the engine’s power with a power split device that is a planetary gearset to send power to both the drive shaft and the generator. The generator produces electricity. Some of this electricity is used to drive the motor and the remainder is stored in the battery [30]. An inverter is used to draw/store power from the battery. [27]

![THS Configuration](image)

*Figure 3.1: THS Configuration. [3]*

The THS transmission consists of the power split device, the generator, the motor, and reduction gears. A schematic of the Toyota Hybrid System can be seen in Figure 3.1.

The system is electronically controlled in order to achieve a variable transmission. It changes to adapt itself to different driving conditions with the aim of working at its most efficient range of operation. The THS vehicle achieves smooth acceleration and deceleration, as well as excellent response [30]. Figure 3.2 shows the system operation depending on the driving conditions.
This system also includes fail-safe functions to ensure that the vehicle does not encounter danger in case of a system malfunction. For further information about the fail safe system consult [30].

*Figure 3.2: System Operation [4]*
**Planetary Gears**

The planetary gearset used for the Prius is commonly known as a power divider. This is because the gasoline engine shaft is linked to the planetary carrier and transmits the motive power to the drive shaft and generator through the ring gear and sun gear respectively. That means that the power is divided, a part is transmitted through a mechanical path and the other through an electrical path. The planetary used for the Prius can be seen in Figure 3.1. [30]

*Figure 3.3: Planetary Gear Prius. [1]*

The THS has proved to be a system which improves fuel-efficiency and reduces CO₂ emissions. This was done by incorporating a high-efficiency gasoline engine, electric vehicle power systems based on nickel-metal hybrid batteries, advanced automatic transmissions, and precision power control. It must be mentioned that the THS system has evolved to the THS II, which is an improved version of the THS. It has the
same transmission but with improved features. For more information about the THS II consult [4].

Section 3.02  Hybrid Power Train (HPT) System

The mechanical engineering department has developed a small-scale hybrid power train for educational purposes. This HPT is a bench-scale “copy” of the Toyota Hybrid System (THS). It must be mentioned that this system was built at the university’s shop in the engineering building. Figure 3.4 shows an overhead view of the HPT transmission. Figure 3.5 depicts the system configuration for the transmission.

Figure 3.4: HPT Overhead View
The system has two Hall-effect tachometers which measure the speed of the air engine and electric motor. Overall control of the system is achieved using a microcontroller, in this case an Arduino Mega 2560. Finally, the system has a “load box” with the aim of simulating the up and down grades of a road.

The HPT is very similar to the THS. The main difference is that instead of using a gasoline engine, it has an air engine powered by the shop’s compressed air at 120 psi (8.3 bar). The flow of air is control by six solenoid valves which are not shown; these will be described with more detail in Chapter 10. Also, it has a DC electric motor and DC generator; so there is no need of an inverter. The system also includes a planetary gearset; in this case, a differential gear since it was cheaper to build this type of geartrain in an educational environment. The system has two Hall-effect tachometers which measures the speed of the air and electric motor. The power control of the vehicle is achieved using

---

**Figure 3.5: HPT System Configuration**
a microcontroller, in this case an Arduino Mega 2560. Finally, the system has a “load box” with the aim of simulating the up and down grades of a road. Figure 3.6 depicts the power and electric paths of the HPT.

Figure 3.6: Power and Electrical Paths HPT

The main difference between the THS and the Rowan HPT is the arrangement of the planetary gearset: the inputs and the output of the planetary gears are not linked to the same shafts as the THS. The rotational shaft of the sun I gear is directly linked to the electric motor and the ring/sun II is directly linked to the air motor. The output, which is the rotational shaft of the planetary carrier inside the gear mechanism, is directly linked to the wheels and generator.
The main difference between the HPT and the THS can be seen in the following table:

*Table 3.1: Differences between THS and HPT*

<table>
<thead>
<tr>
<th></th>
<th>THS</th>
<th>HPT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Power Source</strong></td>
<td>Gasoline Engine</td>
<td>Air Engine</td>
</tr>
<tr>
<td><strong>Electric Motor</strong></td>
<td>AC</td>
<td>DC</td>
</tr>
<tr>
<td><strong>Generator</strong></td>
<td>AC</td>
<td>DC</td>
</tr>
<tr>
<td><strong>Planetary Gears</strong></td>
<td>Sun/ring/planet</td>
<td>Differential</td>
</tr>
<tr>
<td><strong>Sun</strong></td>
<td>Linked to generator</td>
<td>Linked to motor</td>
</tr>
<tr>
<td><strong>Ring</strong></td>
<td>Linked to motor/wheels</td>
<td>Linked to engine</td>
</tr>
<tr>
<td><strong>Carrier</strong></td>
<td>Linked to engine</td>
<td>Linked to gen/wheels</td>
</tr>
<tr>
<td><strong>Inverter</strong></td>
<td>Needed</td>
<td>Not needed</td>
</tr>
<tr>
<td><strong>Load Box</strong></td>
<td>Not needed</td>
<td>Needed</td>
</tr>
</tbody>
</table>
Chapter 4

Planetary gears

The purpose of the chapter is to give a mathematical model for the planetary gearset that can be used in the overall simulation of the system. More specifically, it includes the kinematic and dynamic analysis of the planetary gears.

Section 4.01 Planetary Gears Theory

An epicyclic or planetary gear train consists of one or more rotating gears revolving around a central gear [31]. Hence, the planetary gear is named after the similar behavior of the planets revolving around the Sun in the solar system. It consists of four major components: the sun, one or more planets, a ring gear and a carrier (often called the arm or the spider). An example of a planetary gearset can be seen in Figure 4.1.

Figure 4.1: Planetary Gear Train

The planetary gearset has two inputs and one output that can be freely chosen. This characteristic makes the planetary very versatile and desirable for many
applications. According to [32], planetary gearsets are widely utilized in automotive automatic transmissions and in aerospace drives such as turbine engine reduction gears or helicopter transmissions. At the present time, planetary gears are also used as power splitters in hybrid vehicles (HV). Specifically, Toyota Prius has installed this device in its power train.

Since the Rowan HPT is modeled after the power train of the Toyota Prius, a planetary gear set is also installed. This gear train plays a very important role as mentioned since it has the capability of having two inputs and one output. The two inputs are the main power sources of the system: an electric motor and an air engine. The output is a generator simulating wheels.

They type of planetary gears built was a differential gear. The main difference between the other types is that they are composed of miter gears rather than spur or helical gears. This gear train is often used to measure the difference in speeds between the two inputs for the purpose of synchronization. It was also selected since it is cheaper for students to build it in an educational environment.

The differential gear has two suns, which are those that do not undergo the epicyclic motion experienced by the two planets. Figure 4.2 depicts a schematic of the differential gear and its respectively components.
It can be seen that this device is comprised by two suns, a carrier and two planets. In this case, the second sun plays the same role as the ring. The rotational shaft of the sun gear 1 and sun gear 2 are directly linked to the electric and air motor respectively. They are independent of each other and their motion is combined to create the motion of the carrier which is the output. The output, which is the rotational shaft of the planetary carrier inside the gear mechanism, is directly linked to the wheels and generator. The following table shows the shaft linkages relation.

*Table 4.1: Rotational shaft linkages of the planetary gear*

<table>
<thead>
<tr>
<th>I/O</th>
<th>Components</th>
<th>Linkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input 1</td>
<td>Sun 1 ($S_1$)</td>
<td>Electric Motor</td>
</tr>
<tr>
<td>Input 2</td>
<td>Sun 2 ($S_2$)</td>
<td>Air Motor</td>
</tr>
<tr>
<td>Output</td>
<td>Carrier ($C$)</td>
<td>Wheels and Generator</td>
</tr>
</tbody>
</table>

*Figure 4.2: Schematic of a Differential Gear*
a) **Kinematic Analysis**

The differential gears are arranged in such a way that the carrier gives the average speed of the two sun gears as followed:

\[ \omega_p = \frac{\omega_{s1} + \omega_{s2}}{2} \]  

(4.1)

The angular velocity of the planet gear is the difference between the two sun gears

\[ \omega_p = \frac{\omega_{s1} - \omega_{s2}}{2} \]  

(4.2)

*Figure 4.3: Planetary Gear used for the Project*
(b) Dynamic Analysis

Table 4.2: Syntax of the Components for the Dynamic Analysis

<table>
<thead>
<tr>
<th>Name</th>
<th>Syntax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun 1</td>
<td>$S_1$</td>
</tr>
<tr>
<td>Sun 2</td>
<td>$S_2$</td>
</tr>
<tr>
<td>Carrier</td>
<td>$C$</td>
</tr>
<tr>
<td>Planets</td>
<td>$p$</td>
</tr>
<tr>
<td>Force</td>
<td>$F$</td>
</tr>
<tr>
<td>Angular Velocity</td>
<td>$\omega$</td>
</tr>
<tr>
<td>Torque</td>
<td>$T$</td>
</tr>
<tr>
<td>Inertia</td>
<td>$J$</td>
</tr>
<tr>
<td>Distance</td>
<td>$d$</td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
</tr>
<tr>
<td>Number of planets</td>
<td>$N_p$</td>
</tr>
</tbody>
</table>

(i) Sun Gears

A free-body diagram of the two sun gears is shown in Figure 4.4, with arrows indicating positive directions. The planet gear creates forces upon the two sun gears, and summing torques for each gear gives

Figure 4.4: Free-body Diagram Sun Gears
\[
\sum T = J_{s1} \dot{\omega}_{s1} = T_{s1} - \frac{F_{s1} d_s}{2} \quad (4.3)
\]
\[
\sum T = J_{s2} \dot{\omega}_{s2} = T_{s2} - \frac{F_{s2} d_s}{2} \quad (4.4)
\]

Figure 4.5: Dimension used for the Model

\(d_s\) is the average pitch diameter and \(\frac{d_s}{2}\) is the distance between the point where the force is being applied and the sun gear’s axis of rotation. The same logic applies for the other gears.

Solving Equation (4.3) and (4.4) for the unknown forces gives

\[
F_{s1} = (T_{s1} - J_{s1} \dot{\omega}_{s1}) \frac{2}{d_s} \quad (4.5)
\]
\[
F_{s2} = (T_{s2} - J \dot{\omega}_{s2}) \frac{2}{d_s} \quad (4.6)
\]
(ii) Planet Gears

Figure 4.6: Free-body Diagram Planetary Gears

Summing forces on the planet gear gives

$$\sum F = N_p m_p \ddot{x}_p = F_{s1} + F_{s2} + F_c$$

(4.7)

$$\ddot{x}_p = \frac{d_s}{2} \omega_c$$

(4.8)

Where \(N_p\) is the number of planets in the gear set and \(\ddot{x}_p\) is the tangential acceleration of the center of a planet gear as it revolves about the central axis.

Substituting this into the equation of motion:

$$N_p m_p \frac{d_s}{2} \omega_c = F_{s1} + F_{s2} + F_c$$

(4.9)

Next, we sum the torque on the axis of the planet gear:

$$\sum T = N_p J_p \ddot{\omega}_p = \frac{F_{s1} d_p}{2} - \frac{F_{s2} d_p}{2}$$

(4.10)
\[ N_p I_p \dot{\omega}_p = (F_{s1} - F_{s2}) \frac{d_p}{2} \] \hspace{1cm} (4.11)

(iii) Carrier

\[ J_c \dot{\omega}_c = T_c - \frac{F_c d_s}{2} \] \hspace{1cm} (4.12)

The carrier is attached to the “load box” and the generator, and their combined torque is designated \( T_c \). The carrier force, \( F_c \), is the tangential component of the bearing load of the planet gear. Summing torques on the carrier gives

Here, \( J_c \) is the moment of inertia of the carrier plus the number of planets \( i_p \) multiplied by the moment of inertia of the planet gear (as a lumped mass) as it rotates about the central axis.
Section 4.02  Simplifying the Model

Combining Equations (4.3, 4.4, 4.9, 4.11, 4.12) gives:

\[ m_{sc1} \dot{\omega}_{s1} + m_{sc2} \dot{\omega}_{s2} = T_c + T_{s1} + T_{s2} \]  \hspace{1cm} (4.14)

\[ m_{sp1} \dot{\omega}_{s1} - m_{sp2} \dot{\omega}_{s2} = T_{s1} - T_{s2} \]  \hspace{1cm} (4.15)

where

\[ m_{sc1} = \frac{i_p m_p d_s^2}{8} + \frac{J_c}{2} + J_{s1} \]  \hspace{1cm} (4.16)

\[ m_{sc2} = \frac{i_p m_p d_s^2}{8} + \frac{J_c}{2} + J_{s2} \]  \hspace{1cm} (4.17)

\[ m_{sp1} = \frac{i_p l_p d_s}{2d_p} + J_{s1} \]  \hspace{1cm} (4.18)

\[ m_{sp2} = \frac{i_p l_p d_s}{2d_p} + J_{s2} \]  \hspace{1cm} (4.19)

If we combine Equations (4.14, 4.15) we may solve for the angular accelerations of the sun gears.

\[ \dot{\omega}_{s1} = \frac{T_{s1}(m_{sp2} + m_{sc2}) + T_{s2}(m_{sp2} - m_{sc2}) + T_c m_{sp2}}{m_{sc1} m_{sp2} + m_{sc2} m_{sp1}} \]  \hspace{1cm} (4.20)

\[ \dot{\omega}_{s2} = \frac{T_{s1}(m_{sp1} - m_{sc1}) + T_{s2}(m_{sp1} + m_{sc1}) + T_c m_{sp1}}{m_{sc1} m_{sp2} + m_{sc2} m_{sp1}} \]  \hspace{1cm} (4.21)
If we assume that the planet gears are massless relative to the rest of the system, the equations above simplify to:

\[ \dot{\omega}_{s1} = \frac{(4J_{s2} + J_c)T_{s1} - J_c T_{s2} + 2J_{s2}T_c}{J_c(J_{s1} + J_{s2}) + 2J_{s1}J_{s2}} \]  
(4.22)

\[ \dot{\omega}_{s2} = \frac{-J_cT_{s1} + (4J_{s1} + J_c)T_{s2} + 2J_{s1}T_c}{J_c(J_{s1} + J_{s2}) + 2J_{s1}J_{s2}} \]  
(4.23)

Example: Assuming that \( T_{s1} = T_{s2} = T_s \) and solving equation 4.22 and 4.23 gives that \( \dot{\omega}_{s1} = \dot{\omega}_{s2} \).

\[ \dot{\omega}_{s1} = \frac{(4J_{s2} + J_c)T_s - J_c T_s + 2J_{s2}T_c}{J_c(J_{s1} + J_{s2}) + 2J_{s1}J_{s2}} = \dot{\omega}_{s2} = \frac{(4J_{s1} + J_c)T_s - J_c T_s + 2J_{s1}T_c}{J_c(J_{s1} + J_{s2}) + 2J_{s1}J_{s2}} \]

**Section 4.03  Some Additional Notes**

The sun gears are connected to the shafts for the air and electric motor through a geartrain of ratio \( r = 1.4 \). The speed of the electric motor and air motor are:

\[ \omega_{EM} = r\omega_{s1} \]  
(4.24)

\[ \omega_{AM} = r\omega_{s2} \]  
(4.25)

Finally, the load torque is connected to the carrier through a chain drive with ratio \( r_c = 1 \). The torque seen by the carrier is:

\[ T_c = r_c T_L \]  
(4.26)

where \( T_L \) is the torque provided by the load box.
Chapter 5

Electric Motor

This Chapter describes the mathematical model of the electric motor. Furthermore, it includes how the mathematical constants where found using an experimental approach. Finally, the behavior of the electric motor used for the project is determined and included to the Hybrid Power Train (HPT) mathematical model.

The electric motor is one of the two devices in the system that provides a rotational power that is used to move the wheels/output-shaft. It is electrical because it needs a certain amount of current and a voltage to function.

Section 5.01 Electric Motor Theory

The electric motor is a permanent-magnet DC motor. Electrically, it can be modeled as a series inductance $L_{EM}$ and a resistance $R_{EM}$. A voltage $v_{EM}$, is applied across the terminals of the motor and a current $i_{EM}$ flows through the armature coils. Figure 5.1 depicts the model of the DC motor used for the hybrid power train.

![Theoretical DC Motor](image)

*Figure 5.1: Theoretical DC Motor*
The electric motor produces a torque $T_{EM}$, which is proportional to the current $i_{EM}$ flowing through the coils.

$$ T_{EM} = Ki_{EM} \quad 5.1 $$

Where $K$ is the motor-torque constant (N-m/Amp). Also, as the motor spins it produces a voltage across the coils, like a generator. This voltage opposes the flow of current through the motor; thus it is called back-emf (back electromotive force). The amount of back-emf produced by the spinning motor is proportional to its rate of rotation.

$$ v_e = K_b \dot{\theta} \quad 5.2 $$

Where $K_b$ is the back-emf constant (Volt-s/rad). The spindle of the motor has a rotary inertia $J_{EM}$, and a rotary damping $b_{EM}$.

The Laplace-transformed loop equation of the motor circuit can be written with the voltage as follows:

$$ V_{EM} - L_{EM}sI_{EM} - RI_{EM} - K_b\Theta s = 0 \quad 5.3 $$

where $\Theta$ is the rotation of the motor in the Laplace domain.

Substituting Equation (5.3) and solving for the current $I_{EM}$ gives:

$$ I_{EM} = \frac{V_{EM} - K_b\Theta s}{L_{EM}s + R_{EM}} \quad 5.4 $$

Also, the system has an external torque $\frac{T_{s1}}{r_1}$ supplied by the sun gear 1 of the planetary gearset. The electric motor has to overcome this torque $\frac{T_{s1}}{r_1}$ to rotate. The rotational equation of motion for the spindle system is given by the following equation.

$$ \sum M = J_{EM} \ddot{\theta} = T_{EM} - b_{EM} \dot{\theta} - \frac{T_{s1}}{r_1} \quad 5.5 $$

Or, rearranging to place the torque inputs on the LHS of the equation:
It must be mentioned that each motor, depending on its characteristics, has a different value for the constants $J_{EM}$ and $b_{EM}$. Figure 5.2 depicts the electric motor used for the small-scale hybrid power train (Model: MY6812 Unite Motor). It has a maximum capacity of 24V and 135W.

\[ T_{EM} - \frac{T_{s1}}{r_1} = J_{EM} \ddot{\theta} + b_{EM} \dot{\theta} \]  \hspace{1cm} 5.6

The next section will describe the methods used to find the rotary inertia and damping coefficient of the electric motor used in the hybrid powertrain.

**Section 5.02 Calculating Electric Motor Constants**

In order to find the constants of the motor used in the hybrid powertrain, the external torque from the planetary gearset, $T_{s1}$, is omitted.

\[ T_{EM} = J_{EM} \dot{\theta} + b_{EM} \dot{\theta} \]  \hspace{1cm} 5.7
It must be mentioned that for the electric motor has mechanical and electrical motor constants that need to be measured in order to determined its behavior.

a) **Measuring the Mechanical Motor Constants ($b_{EM}$ and $J_{EM}$)**

In measuring the mechanical motor constants the motor leads are left disconnected; thus the electric motor torque, $T_{EM}$, is zero. Next, a known external torque, $T_{ext}$, is applied to the motor, and its resulting motion is measured.

(i) **Methodology**

The equation of motion for the motor without electrical input is

$$T_{ext} = J_{EM} \ddot{\theta} + b_{EM} \dot{\theta} \tag{5.8}$$

where $T_{ext}$ is an externally-applied torque, $J_{EM}$ is the rotary inertia of the rotating part of the motor and $b_{EM}$ is the rotational damping. It is customary in the analysis of motors to be concerned with angular velocity, $\omega$, rather than angular displacement, $\theta$.

$$T_{ext} = J_{EM} \dot{\omega} + b_{EM} \omega \tag{5.9}$$

If it is assumed that the external torque is applied as a step input and the initial speed is equal to zero then the solution to this first-order differential equation is

$$\omega(t) = \frac{T_{ext}}{b_{EM}} \left(1 - e^{-\frac{t}{J_{EM}}} \right) \tag{5.10}$$

For large $t$, the speed approaches a maximum

$$\omega(\infty) = \omega_{max} = \frac{T_{ext}}{b_{EM}} \tag{5.11}$$

and solving for $b_{EM}$ gives

$$b_{EM} = \frac{T_{ext}}{\omega_{max}} \tag{5.12}$$
We can rearrange Equation (5.10) to solve for $J_{EM}$

$$J_{EM} = \frac{t_i b_{EM}}{\ln \left( \frac{\omega_{max}}{\omega_{max} - \omega_i} \right)}$$

where $\omega_i$ is the angular velocity at the time step $t_i$. This equation can be solved for all time steps until $\omega_{max}$ is reached, with the average taken as the correct value for $J_{EM}$.

(ii) Experimental approach

The rotary inertia $J_{EM}$ and damping $b_{EM}$ are calculated experimentally. Taking into account that these are constants, they can be calculated by applying a known external torque $T_{ext}$ and measuring the angular velocity $\dot{\omega}$ of the electric motor.

![Figure 5.3: Pulley-Mass System](image)

The known torque $T_{ext}$ is applied by using a pulley-mass system as shown in Figure 5.3. As the mass $m$ falls it applies a tangential force $F$ to the shaft of the electric
motor. In consequence the motor rotates generating a rotational velocity \( \dot{\theta} \). This \( F \) is calculated using Newton’s second law as followed:

\[
F = mg
\]  

5.14

The \( T_{ext} \) is calculated using the following equation:

\[
T_{ext} = Fd
\]  

5.15

Where \( d \) is the distance between the center of the shaft and the place where the force has been applied. The experiment was repeated four times.

Figure 5.4 shows the plot after keeping track of \( \dot{\theta} \) during the experiment. The plot represents the average angular velocity. The values used for the experimental approach were \( m = 0.225 \text{ Kg}; = 0.0207 \text{m} \). The gravity was selected as \( g = 9.81 \frac{m}{s^2} \) since the experiment was done at Rowan University, Glassboro which is close to the coast. After substituting these values in Equation (5.14) and (5.15) the value obtained for \( T_{ext} \) is 0.0457 \( \text{Nm} \).
• Calculating $b_{EM}$
  The constant $b_{EM}$ was determined by Equation (5.12). It must be mentioned that when $\omega$ is constant, $\dot{\omega} = 0$. Hence, $b_{EM} = \frac{T_{ext}}{\omega}$

  In Figure 5.4 it can be seen that this occurs after 2.17 s. The value of the damping constant $b_{EM}$ for the electric motor is $2.3594E-03 \text{ Nms}$.

• Calculating $J_{EM}$
  Now that we obtained $b_{EM}$, the constant $J_{EM}$ is calculated by substituting the values in Equation (5.13). The value of the Inertia $J_{EM}$ for the electric motor is $1.0460E-03 \text{ kgm}^2$.

(iii) Verification

The inertia and damping constants of the motor found during the previous experiment are inserted in Equation (5.10). This is done for verification purposes. In order to assure that the values found for the constants describe the behavior of the actual motor. Figure 5.5 depicts the theoretical vs. the real-motor/experimental curves.
It can be noticed that the curves are very similar hence the values of $J_{EM}$ and $b_{EM}$ found in the previous section are accepted.

b) **Measuring the Electrical Motor Constants $K_m$ and $T_m$**

**(i) Methodology**

Next, the behavior of the motor is measured when a voltage is applied to the coils, but without external torque. Performing a voltage loop analysis around the motor circuit (and neglecting the inductance of the coils) gives

$$v_{EM} - i_{EM}R_{EM} - v_b = 0$$ \hspace{1cm} 5.16

Solving for the motor current gives

$$i_{EM} = \frac{1}{R_{EM}}(v_{EM} - v_b)$$ \hspace{1cm} 5.17

But the electrical torque is related to current through the motor-torque constant as showed in Equation (5.1):

$$T_{EM} = K_i_{EM}$$

and the back-emf through the back-emf constant as shown in Equation (5.2)

$$v_b = K_b\omega$$

Substituting these into the equation of motion, Equation (5.7), and replacing the external torque by the electrical torque, $T_{EM}$, gives

$$\frac{K}{R_{EM}}(v_{EM} - K_b\omega) = J_{EM}\dot{\omega} + b_{EM}\omega$$ \hspace{1cm} 5.18

Finally, placing this equation into standard form gives

$$K_m v_{EM} = T_m\dot{\omega} + \omega$$ \hspace{1cm} 5.19

Where
\[ K_m = \frac{K}{b_{EM}R_{EM} + K_bK} \quad 5.20 \]
\[ T_m = \frac{J_{EM}R_{EM}}{b_{EM}R_{EM} + K_bK} \quad 5.21 \]

\( K_m \) is the motor gain constant and \( T_m \) is the electrical time constant. The solution to this equation is similar to Equation (5.10).

\[ \omega(t) = K_m v_{EM} \left( 1 - e^{-\frac{t}{T_m}} \right) \quad 5.22 \]

\( K_m \) and \( T_m \) can be measured in a similar fashion to \( J_{EM} \) and \( b_{EM} \). First, note that the maximum speed of the motor (without load) is

\[ \omega_{max} = K_m v_{EM} \quad 5.23 \]

so that

\[ K_m = \frac{\omega_{max}}{v_{EM}} \quad 5.24 \]

And at any time step \( t_i \), \( T_m \) can be solved as follows:

\[ T_m = \frac{t_i}{\ln \left( \frac{\omega_{max}}{\omega_{max} - \omega_i} \right)} \quad 5.25 \]

(ii) Experimental approach

A step input \( v_{EM} = 24V \) was applied to the electric motor and the speed was recorded using a tachometer. Substituting the voltage into Equation (5.22) gives:

\[ \omega(t) = 24K_m \left( 1 - e^{-\frac{t}{T_m}} \right) \quad 5.26 \]

The plot of the experimental data obtained after applying a step input of 24V can be seen in Figure 5.6. Eight trials were done.
Figure 5.6: Step Response of Electric Motor with 24V Applied

- **Calculating $K_m$**

  It must be noted that when the time $t$ tends to $\infty$, the term $e^{-\frac{t}{\tau_m}} = 0$; in consequence $\omega(\infty) = \omega_{max} = v_{EM}K_m = 24K_m$.

  As seen in Figure 5.6 the average angular velocity when $t$ tends to $\infty$ is $\omega_{max} = 255.51 \text{ rad/s}$. The proportional constant $K_m$ between voltage input and speed output value for a unit step is found by substituting the values in Equation (5.24), which gives:

  $$K_m = \frac{255.51 \text{ rad/s}}{24 \text{ V}} = 10.6459 \frac{\text{ rad}}{\text{ Volt} - \text{ sec}}$$

- **Calculating $T_m$**

  Now that we obtained $K_m$, the constant $T_m$ is calculated using Equation (5.25). It is important to mention that this value is calculated while the acceleration $\neq 0$; that is,
while the motor has not reached a steady state. As seen in Figure 5.6 this occurs for the electrical motor approximately from 0 until 2.2 seconds. The value of the constant $T_m$ found after substituting the experimental data and averaging the results is 0.038448261 s.

![Velocity Electrical Motor](image)

*Figure 5.7: Behavior of the Electric Motor: Experimental vs. Theoretical*

(iii) Verification

The values found for $K_m$ and $T_m$ are inserted into Equation (5.22) in order to get the theoretical velocity and verify that the values obtained resemble the real motor behavior. Figure 5.7 depicts the experimental and the theoretical velocity of the motor. It can be noted that their behavior is very similar hence the constant values obtained for $K_m$ and $T_m$ are accepted. The max error in the data is at approx. 2.3 seconds and the max error percentage of the data is 11.1% at 0.65 seconds.
Calculating $R_{EM}$, $K$ and $K_b$

The resistance of the coil $R_{EM}$ is determined by measuring the electric motor’s resistance with a digital multimeter. The value obtained for $R_{EM}$ was $18.0303 \, \Omega$. Also the torque constant $K$ and the back emf constant $K_b$ can be found by solving Equations (5.20) and (5.21) and substituting the previously found values of $J_{EM}, K_m, R_{EM}$ and $T_m$ that can be found in Table 5.1. As a note, it must be mentioned that the resistance was measure when the electric motor wasn’t hot.

\[ K = \frac{J_{EM} K_m R_{EM}}{T_m} \]  

\[ K_b = \frac{J_{EM} - T_m b_{EM}}{J_{EM} K_m} \]  

The values obtained are $K = 6.412268 \, Nm/Amp$ and $K_b = 8.7340 \times 10^{-2} \, Volt \, s/rad$. Table 5.1 depicts the values obtained for the electric motor used for the HPT.

<table>
<thead>
<tr>
<th>Constants</th>
<th>Abbreviation</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Damping</td>
<td>$b_{EM}$</td>
<td>$2.3447 \times 10^{-3}$</td>
<td>$N \cdot m \cdot s$</td>
</tr>
<tr>
<td>Rotary Inertia</td>
<td>$J_{EM}$</td>
<td>$1.2844 \times 10^{-3}$</td>
<td>$kg \cdot m^2$</td>
</tr>
<tr>
<td>Motor Gain constant</td>
<td>$K_m$</td>
<td>10.6459</td>
<td>rad/(V·s)</td>
</tr>
<tr>
<td>Motor Time Constant</td>
<td>$T_m$</td>
<td>0.038448</td>
<td>s</td>
</tr>
<tr>
<td>Resistance of the Coil</td>
<td>$R_{EM}$</td>
<td>18.0303</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>$K$</td>
<td>6.412268</td>
<td>N·m/A</td>
</tr>
<tr>
<td>Back emf constant</td>
<td>$K_b$</td>
<td>$8.7340 \times 10^{-2}$</td>
<td>V·s/rad</td>
</tr>
</tbody>
</table>
(d) Calculating $T_{EM}$ and the $v_e$ (back emf)

Finally, the torque of the motor $T_{EM}$ can be calculated substituting the torque constant value found previously in Equation (5.1). Similarly the back emf $v_e$ is calculated using Equation (5.2)

$$T_{EM} = 6.412268 \ i_{EM} \ Nm$$  

Also, the equation obtained for the back electromotive force $v_e$ is:

$$v_e = 8.7340 * 10^{-2} \ \omega \ \ V$$

Section 5.03 Including the Electric Motor in the HPT System

Now that the behavior of the motor has been determined it can be incorporated into the system model. In other words, the torque supplied by the planetary gears is included. As seen in Figure 5.1 the torque supplied by the sun gear 1 $\ \frac{T_{s1}}{r}$, is added to the calculations since the electric motor is connected to this shaft. It must be noted that it is divided by a ratio $r_1$ because this torque is supplied to the shaft after passing through the geartrain of ratio $r_1$.

Including the torque supplied by the sun gear 1, $\ \frac{T_{s1}}{r}$, into the equation of motion, Equation (5.18) gives:

$$\frac{K}{R_{EM}} (v_{EM} - K_b \omega) = j_{EM} \omega_{EM} + b_{EM} \omega_{EM} - \frac{T_{s1}}{r_1}$$

Solving this for the torque on the sun gear gives:

$$\frac{T_{s1}}{r_1} = \frac{K}{R_{EM}} \ v_{EM} - \left( \frac{KK_b}{R_{EM}} + b_{EM} \right) \ \omega_{EM} - j_{EM} \omega_{EM}$$

It must be mentioned that the motor speed $\omega_{EM}$ is the same as the sun gear 1 speed $\omega_{s1}$ multiplied by the gear ratio.
\[ \omega_{EM} = \omega_s r_1 \quad 5.33 \]

Substituting this and solving the equation for \( T_{s1} \) gives:

\[ T_{s1} = \left[ \frac{K}{R_{EM}} v_{EM} - \left( \frac{KK_b}{R_{EM}} + b_{EM} \right) r_1 \omega_s \right] r_1 - J_{EM} r_1^2 \omega_{s1} \quad 5.34 \]

The moment of inertia of the motor \( J_{EM} r_1^2 \) has been already taken into account in the planetary gears analysis. This was done since it is easiest to transfer the moment of inertia of the motor directly to the sun gear shaft, through the gear ratio.

\[ J_{s1} = J'_{s1} + J_{EM} r_1^2 \quad 5.35 \]

As mentioned in the planetary gear section \( J'_{s1} \) is the moment of inertia of the sun gear shaft and sun gear alone. If the inertia of the electric motor is accounted in this way, then we must modify the torque equation as follows:

\[ T_{s1} = \left[ \frac{K}{R_{EM}} v_{EM} - \left( \frac{KK_b}{R_{EM}} + b_{EM} \right) r_1 \omega_s \right] r_1 \quad 5.36 \]

Finally by substituting the values previously found for this particular system shown in Table 5.1 and including \( r_1 = 1.4 \) gives:

\[ T_{s1} = (0.49047481 v_{EM} - 0.06456891 \omega_{s1}) Nm \quad 5.37 \]
Chapter 6

Air Engine

This chapter includes the equations that describe the behavior of an air engine and how the model is incorporated into the overall system model. Furthermore, it describes the method used to find the optimal speed of the engine. Finally, how the Air Engine is included to the hybrid power train system.

Section 6.01 Air Engine Theory

The air engine is the second device that provides a mechanical rotational motion with the aim of moving the output shaft/wheels. This motor is powered by compressed air at 120 psi supplied by an air line in the shop. In order to control the different speeds of the motor, the air flow $q$, needs to be regulated.

Figure 6.1: Schematic Diagram of Air Flow Control Module

The air engine is the second device that provides a mechanical rotational motion with the aim of moving the output shaft/wheels. This motor is powered by compressed air at 120 psi supplied by an air line in the shop. In order to control the different speeds of the motor, the air flow $q$, needs to be regulated.
As seen in Figure 6.1 the air is regulated by supplying it into an aluminum block with six different diameter size holes in order to choke the flow of air. The exhaust is directed into a set of six solenoid valves. Finally, the air from the valves is combined and sent to the Air Engine.

By opening and closing each solenoid valve, the speed of the air engine can be regulated. The electrical connection of these valves was planned so that the six valves can work simultaneously in a binary pattern. In consequence, a range of 63 different combinations can be obtained generating different airflows.

Chapter 10, contains more detailed information about the Air Flow Control Module: The Hardware Implementation of the Control System.

Figure 6.2: Theoretical Air Engine

Figure 6.2 depicts a model of the air Engine. The air engine produces a torque \( T_{AM} \), which is proportional to the airflow \( q \) and air pressure. Similarly to the electric motor, the spindle of the motor has a rotational inertia \( J_{AM} \) and a rotary damping \( b_{AM} \). Also, this system has an external torque \( T_{2} \) supplied by the sun gear 2 shaft, after passing through the geartrain of ratio \( r_2 \).
The rotational equation of motion for the spindle system is the same as Equation (5.6) but applied for the air engine as follows:

\[ T_{AM} - \frac{T_s}{r_2} = J_{AM} \ddot{\theta}_{AM} + b_{AM} \dot{\theta}_{AM} \]  \hspace{1cm} (6.1)

To analyze the air engine in isolation, we neglect the torque supplied by the sun gear 2, as in Equation (5.7):

\[ T_{AM} = J_{AM} \ddot{\theta}_{AM} + b_{AM} \dot{\theta}_{AM} \]  \hspace{1cm} (6.2)

Also, if the external torque is assumed to be applied as a step input and the initial speed equal to zero, then the solution to this first-order differential equation is the same as Equation (5.10) but applied for the air engine:

\[ \omega(t) = \frac{T_{ext}}{b_{AM}} \left(1 - e^{-\frac{b_{AM}}{J_{AM}}}\right) \]  \hspace{1cm} (6.3)

Applying the same logic as Equation (5.11-13) gives:

\[ \omega(\infty) = \omega_{max} = \frac{T_{ext}}{b_{AM}} \]  \hspace{1cm} (6.4)

and solving for the frictional damping in the air engine:

\[ b_{AM} = \frac{T_{ext}}{\omega_{max}} \]  \hspace{1cm} (6.5)

Rearranging Equation (6.3) to solve for rotary inertia:

\[ J_{AM} = \frac{t_i b_{AM}}{\ln \left( \frac{\omega_{max}}{\omega_{max} - \omega_i} \right)} \]  \hspace{1cm} (6.6)
where $\omega_i$ is the angular velocity at the time step $t_i$.

The air engine used for the HPT system is shown in Figure 6.3.

![Air engine used for the small-scale Power Train System](image)

*Figure 6.3: Air engine used for the small-scale Power Train System*

**Section 6.02  Experimental Approach to determine the Air Engine Parameters**

(a) **Calculating $b_{AM}$ and $J_{AM}$**

The rotary inertia $J_{EM}$ and damping $b_{EM}$ are calculated experimentally, following the same procedure as explained in Chapter 5.

Using the same values $m = 0.225$ kg; $d = 0.020701$ m; $g = 9.81 \frac{m}{s^2}$, and $T_{known}$ is 0.04569228 Nm. Figure 6.4 depicts the behavior of the air engine during this experiment.
Figure 6.4: Air Engine Behavior during Experiment

The experiment was conducted four times. Figure 6.4 depicts the average experimental behavior of the air engine.

The experimental results are shown in the following table:

<table>
<thead>
<tr>
<th>Constants</th>
<th>Abbreviation</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Damping</td>
<td>$b_{EM}$</td>
<td>1.08867E-03</td>
<td>N · m · s</td>
</tr>
<tr>
<td>Rotary Inertia</td>
<td>$J_{EM}$</td>
<td>2.84769E-04</td>
<td>kg · m²</td>
</tr>
</tbody>
</table>

(b) Verification

The values found for $b_{AM}$ and $J_{AM}$ are inserted into Equation (6.3) in order to get the theoretical curve of the behavior of the motor and verify that the values obtained resemble the real motor performance. Figure 6.5 depicts the experimental and the theoretical behavior of the motor. It can be noted that the curves are reasonably similar hence the constant values obtained for $b_{AM}$ and $J_{AM}$ are accepted.
Calculating \( T_{AM} \)

In contrast to the torque produced by the electric motor \( T_{EM} \), the torque produced by the air engine \( T_{AM} \) is measured experimentally using a dynamometer. The torque/speed data will be embedded in an interpolative lookup table for the overall system model.

The hardware required to do the testing was the six solenoid valves and the aluminum block in order to regulate the airflow. Also, a flow meter was required to measure the airflow \( q \), at each opening, \( n_i \), \( i = [0, 63] \). Finally, a dynamometer is used to apply and measure different torques \( T_{AM} \) at each opening \( n_i \) and measure the speed \( \dot{\theta} \) of the motor at each circumstance.

An example curve collected during this experiment is shown in Figure 6.6. In this case the orifice block openings are set to \( n = 35 \). In binary this is \( 35 = 0100011 \). That means that for this instance three valves are opened. For more information about the design of the aluminum block consult Chapter 10. The air flow at
this opening is 3.25 CFM. It can be noted that as the torque load increased the speed of the motor decreased.

**Figure 6.6: Angular Velocity Vs Torque. Velocity n=35**

Appendix 1 depicts the experimental data obtained to build the interpolative look-up table using Matlab. Specifically with the table you can find the different speeds $\dot{\theta}$ of the air engine obtained by varying both $n_i$ and $T_{AM}$.

In conclusion, this table is done to determine the air engine’s torque during operation; simply by knowing the operating air engine’s speed $\omega_{AM}$ and its air flow setting $n_i$.

### Section 6.03 Determining the Air Engine Optimal Speed

The optimal speed of a motor is reached when its efficiency is maximum. This occurs when the air engine converts the highest quantity of power provided by the compressed air into a mechanical motion.
The efficiency $\eta$ is calculated as followed:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100$$  \hfill (6.7)

Where $P_{out}$ is the useful power produced by the system, and $P_{in}$ is the power supplied by the compressed air. $P_{out}$ is the work per time unit accomplished by the air engine:

$$P_{out} = T_{AM}\omega$$  \hfill (6.8)

$P_{in}$ is the work per time unit provided by the compressed air

$$P_{in} = qRT\ln\left(\frac{P_1}{P_2}\right)$$  \hfill (6.9)

- $R$: ideal gas constant
- $q$: mass flow rate
- $T$: is the room temperature.
- $P_1$: is the shop line pressure
- $P_2$: is the atmospheric pressure.
- Substituting Equation (6.8) and (6.9) in Equation (6.7) gives:

$$\eta = \frac{T_{AM}\omega_{AM}}{RTqT\ln\left(\frac{P_1}{P_2}\right)} \cdot 100$$  \hfill (6.10)

Mass flow rate can also be calculated by:

$$q = \rho Q_v$$  \hfill (6.11)

Where:

- $\rho$: mass density of the fluid
- $Q_v$: volumetric flow rate of air at high pressure

Substituting Equation (6.11) in (6.10) gives:
\[ \eta = \frac{T_{AM} \omega_{AM}}{RT(pQ_v)\ln \left( \frac{p_1}{p_2} \right)} \cdot 100 \]  

(6.12)

By definition it is also known that:

\[ \rho = \frac{p}{RT} \]  

(6.13)

Substituting Equation (6.13) into (6.12) gives:

\[ \eta = \frac{T_{AM} \omega_{AM}}{RT \left( \frac{p_2}{RT} Q_v \right) \ln \left( \frac{p_1}{p_2} \right)} \cdot 100 \]  

(6.14)

\[ \eta = \frac{T_{AM} \omega_{AM}}{Q_v P_2 \ln \left( \frac{p_1}{P_2} \right)} \cdot 100 \]  

(6.15)

The flow meter that used is XXX. It does not give you directly \( Q_v \), but \( Q_m \) which is the mass flow rate at atmospheric conditions. The flow meter assumes that air is flowing at atmospheric pressure, and reads \( Q_m \). To convert to \( Q_v \) (the actual flow rate at high pressure) a correction factor provided by the manufacture is needed. The correction factor is described below.

According to the manufacture in order to the readings reassemble the reality a correction factor needs to be added. The flow correction is done by changing \( P_2 \) by \( \sqrt{\frac{P_1}{P_2}} \).

For more information about this correction factor consult XX.

\[ \eta = \frac{T_{AM} \omega_{AM}}{Q_m \sqrt{\frac{P_1}{P_2}} P_2 \ln \left( \frac{p_1}{P_2} \right)} \cdot 100 \]  

(6.16)

After doing some algebra the efficiency is calculated as follows:
\[
\eta = \frac{T_{AM}\omega_{AM}}{Q_{m}P_{2}^{1/2}P_{1}^{1/2}\ln\left(\frac{P_{1}}{P_{2}}\right)} \times 100
\] (6.17)

Table 6.2 depicts the values and units of the variables that need to be substitute in Equation (6.17).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop line Pressure</td>
<td>$P_{1}$</td>
<td>914,934.29 Pa</td>
</tr>
<tr>
<td>Atmospheric Pressure</td>
<td>$P_{2}$</td>
<td>101,352.93 Pa</td>
</tr>
<tr>
<td>Torque Air engine</td>
<td>$T_{AM}$</td>
<td>Varies. See Appendix 1 Nm</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>$\omega_{AM}$</td>
<td>Varies. See Appendix 1 rad/s</td>
</tr>
<tr>
<td>Mass flow rate at</td>
<td>$Q_{m}$</td>
<td>Varies. See Appendix 1 m$^3$/s</td>
</tr>
<tr>
<td>atmospheric conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(as given by the flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>meter)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After substituting the values obtained in Equation (6.15), the efficiency of the air engine was calculated. The efficiency values are also included in appendix 1.

![Air engine Efficiency](image)

*Figure 6.7: Air Engine’s Efficiency*
The efficiency of the air engine is calculated using the experimental data collected
in the previous experiment. Figure 6.7 depicts the efficiency of the motor. The legend
represents the openings of the orifice block. The graphs include the openings from 5 to
63 in steps of 5. Beginning in this case with the step 10 = B000110 , since at this opening
the air engine began to move when it was attached to the dynamometer. After getting the
experimental data, the maximum efficiency reached was 4.4% when the speed was
between 84rad/sec (802 RPM) and 104 rad/sec (993 RPM).

Section 6.04 Including the Air Engine to the Hybrid Power Train System

Now that the behavior of the motor has been determined, it can be included to the
system. In other words, as seen in Figure 6.2 the torque supplied by the sun gear 2 is
added to the calculations since the motor is connected to this shaft.

Summing torques on the engine gives

\[ \sum T = J_{AM} \dot{\omega}_{AM} = T_{AM} - b_{AM} \omega_{AM} - \frac{T_{s2}}{r_2} \]  \hspace{1cm} (6.18)

The simplest way to find the torque produced by the engine is to use experimental
data embedded in an interpolative lookup table as mentioned previously (See Appendix
A).

\[ T_{AM} = T_{AM}(\omega_{AM}, n_t) \]  \hspace{1cm} (6.19)

The final equation of motion is then:

\[ T_{s2} = r_2(T_{AM} - b_{AM} \omega_{AM}) - r_2 J_{AM} \dot{\omega}_{AM} \]  \hspace{1cm} (6.20)
It must be mentioned that the motor speed $\omega_{AM}$ is the same as the sun gear 2 speed $\omega_{s2}$, multiplied by the gear ratio $r_2$.

$$\omega_{AM} = \omega_{s2}r_2 \quad (6.21)$$

Substituting this and solving the equation for $T_{s2}$ gives:

$$T_{s2} = r_2(T_{AM} - b_{AM}\omega_{s2}r_2) - J_{AM}r_2^2\dot{\omega}_{s2} \quad (6.22)$$

As with the electric motor, the rotational inertia of the air engine has already been included in the analysis of the planetary gears with the moment of inertia of the sun gear 2 shaft.

$$J_{s2} = J'_{s2} + J_{AM}r^2 \quad (6.23)$$

Finally, the sun gear 2 torque equation becomes

$$T_{s2} = r_2(T_{AM} - b_{AM}\omega_{s2}r_2) \quad (6.24)$$

Substituting the air engine values:

<table>
<thead>
<tr>
<th>Table 6.3: Air engine Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Abbreviation</strong></td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Rotary Damping</td>
</tr>
<tr>
<td>Gear ratio 2</td>
</tr>
<tr>
<td>Torque Air engine</td>
</tr>
</tbody>
</table>

Gives:

$$T_{s2} = 1.4T_{AM} - 2.1337932 \cdot 10^{-3} \omega_{s2} Nm \quad (6.25)$$
Chapter 7

Load Torque from the Generator

This chapter describes the roll of the generator in the system. It includes the equations that describe its behavior and the experimental approach to find the constants used for the mathematical model. Last, it contains the final equations that are used for the model.

![Generator schematic](image)

**Figure 7.1: Generator theoretical model**

**Section 7.01 Generator Theory**

An electrical generator is a rotating electric machine that converts mechanical power into electrical power. A schematic of the generator used for the system can be seen in Figure 7.1. A generator can be modeled as an electric motor loaded in reverse. In the hybrid power train system the generator is responsible for charging the battery that powers the electric motor. When there is no battery, it converts the mechanical power from the air motor to generate a voltage $v_g$ and a current $i_g$.

The generator used for the system can be seen in Figure 7.2. It can be noted that it is the same model as the electric motor (Model: MY6812 Unite Motor); with a maximum capacity of 24V and 135W. The same torque/current relationships derived earlier for the Electric Motor in Section 5.01 can be used to find the load torque produced by the generator.
The dimensions of the electric motor are the following (mm):

![Electric Motor dimension in (mm)](image)

*Figure 7.3: Electric Motor dimension in (mm)*
A theoretical model of the generator is shown in Figure 7.1, with $T_{lg}$ being the portion of the carrier load torque created by the generator. Summing torques on the generator shaft gives

$$
\sum T = J_g \dot{\omega}_g = T_g - b_g \omega_g - \frac{T_{lg}}{r_c}
$$

(7.1)

where

- $T_g$: load torque created when the generator is producing power
- $T_{lg}$: portion of the carrier load torque created by the generator
- $b_g$: frictional damping in the generator
- $i_g$: electrical current in the generator
- $R_g$: resistance in the windings of the generator
- $R_b$: internal resistance in the battery (usually negligible compared to $R_g$)
- $v_g$: open-circuit voltage produced by the generator
- $v_{ba}$: voltage across the terminals of the battery

Note that $T_g$ and $\omega_g$ have opposite sign when the generator is producing power.

There are two possible configurations for the generator

1. The generator is connected to and charging the battery with current $i_g$. For this to be possible, the generator voltage, $v_g$, must be greater than the battery voltage, $v_b$.

2. The generator is disconnected from the battery, and no generator current is flowing. The generator electrical load torque, $T_g$, is zero and the only load created by the generator is through friction.

If the generator is charging the battery, then the electromechanical relations given in Equations (5.1) and (5.2) are used:

$$
T_g = Ki_g
$$

(7.2)
\[ v_g = K_b \omega_g \quad (7.3) \]

Summing voltages around the circuit loop gives
\[ v_g - i_g (R_b + R_g) - v_b = 0 \quad (7.4) \]

Substituting the expressions for current and generator voltage and solving for the generator torque gives
\[ T_g = \frac{K}{R_b + R_g} \left( K_b \omega_g - v_b \right) \quad (7.5) \]

Therefore, charging of the battery is only possible when the speed of the generator is greater than
\[ \omega_g > \frac{v_b}{K_b} \quad (7.6) \]

Substituting the torque from Equation (7.5) into Equation (7.1) and solving for the load torque on the carrier gives
\[ \frac{T_{cg}}{r_c} = \frac{K}{R_b + R_g} v_b - \left( \frac{K K_b}{R_b + R_g} - b_g \right) \omega_g - J_g \dot{\omega}_g \quad (7.7) \]

But the speed of the generator is the same as the carrier multiplied by the chain ratio. Also, we choose to attach the moment of inertia of the generator to the carrier, as with the electric motor.
\[ J_c = J_c' + J_g r_c^2 \quad (7.8) \]
It must be mentioned that the generator speed $\omega_g$ is the same as the carrier speed $\omega_c$ multiplied by the gear ratio.

$$\omega_g = \omega_c r_c$$  \hspace{1cm} (7.9)

Thus, the torque created by the generator is

$$T_{cg} = \left[ \frac{K}{R_b + R_g} v_b - \left( \frac{KK_b}{R_b + R_g} - b_g \right) r_c \omega_c \right] r_c$$  \hspace{1cm} (7.10)

if

$$\omega_g > \frac{v_b}{K_b}$$  \hspace{1cm} (7.11)

or

$$T_{cg} = b_g r_c^2 \omega_c$$  \hspace{1cm} (7.12)

otherwise.

Section 7.02  Experimental approach to find the back-emf constant ($K_b$).

The same values as the electric motor back-emf constant $K_b$ was found solving Equation (7.3) for $K_b$ which gives:

$$K_b = \frac{v_g}{\omega_g}$$

It can be noted that only by measuring the angular velocity and the voltage produced by the generator $K_b$ can be easily found.

Table 7.1 depicts the data obtained after measuring the voltage generated by the generator at different angular velocities.
The average $K_b$ value obtained is $9.609 \times 10^{-2}$ Vs/rad. The generator and the electrical motor’s $K_b$ constants should be the same since the motors are the same type. Comparing both values, it must be noted that even though both back-emf were found using different procedures the value obtained for $K_b$ for the electric motor is $8.7340 \times 10^{-2}$ Vs/rad which is similar to the $K_b$ obtained for the generator. This is another justification that the constants found for the electric motor are correct. It must be mentioned that the error percentage both values is less than 10%. It must be considered that this are off the shelve motors/generators,

### Table 7.1: Data obtained to calculate the Generator’s $K_b$

<table>
<thead>
<tr>
<th>Voltage Generator $v_g$</th>
<th>$\omega_g = rad/sec$</th>
<th>$K_b = V \cdot s/rad$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>28.48377339</td>
<td>0.08776927</td>
</tr>
<tr>
<td>5</td>
<td>50.99852074</td>
<td>0.098042059</td>
</tr>
<tr>
<td>8.5</td>
<td>86.49851773</td>
<td>0.09826758</td>
</tr>
<tr>
<td>13.9</td>
<td>143.5707843</td>
<td>0.096816355</td>
</tr>
<tr>
<td>17.2</td>
<td>178.756622</td>
<td>0.096220212</td>
</tr>
<tr>
<td>22</td>
<td>224.100276</td>
<td>0.098170339</td>
</tr>
<tr>
<td>23.4</td>
<td>240.331838</td>
<td>0.097365377</td>
</tr>
<tr>
<td><strong>Average $K_b$</strong></td>
<td><strong>0.096093027</strong></td>
<td><strong>Vs/rad</strong></td>
</tr>
</tbody>
</table>
Section 7.03 Including the experimental data

Table 7.2: Values obtained for the system that will be substituted in the equation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary Damping b_g</td>
<td>2.3447 \times 10^{-3}</td>
<td>Nms</td>
</tr>
<tr>
<td>Rotary Inertia J_g</td>
<td>1.2844 \times 10^{-3}</td>
<td>kgm^2</td>
</tr>
<tr>
<td>Motor Gain constant K_m</td>
<td>10.6459</td>
<td>rad/(sec-Volt)</td>
</tr>
<tr>
<td>Motor Time Constant T_m</td>
<td>0.038448</td>
<td>s</td>
</tr>
<tr>
<td>Resistance of the Coil R_g</td>
<td>18.0303</td>
<td>Ω</td>
</tr>
<tr>
<td>Resistance of the battery R_b</td>
<td>Neglige</td>
<td>Ω</td>
</tr>
<tr>
<td>Torque Constant K</td>
<td>6.412268</td>
<td>Nm/Amp</td>
</tr>
<tr>
<td>Back-emf constant K_b</td>
<td>9.609 \cdot 10^{-2}</td>
<td>Volt/s/rad</td>
</tr>
<tr>
<td>Battery Voltage v_{bat}</td>
<td>24</td>
<td>Volt</td>
</tr>
<tr>
<td>Voltage generator v_g</td>
<td>9.609 \cdot 10^{-2} \cdot \omega_g</td>
<td>Volt</td>
</tr>
<tr>
<td>Torque generator T_g</td>
<td>6.412268 \cdot i_g</td>
<td>Nm</td>
</tr>
<tr>
<td>Gear Ratio r_c</td>
<td>1</td>
<td>unit less</td>
</tr>
</tbody>
</table>

Substituting these values in equations (7.10), (7.11) and (7.12) gives:

If

\[ \omega_g > 274.7882 \frac{rad}{s} = 2624 \text{ RPM} \]  

\[ T_{cg} = (8.5353 - 0.0287 \omega_c) Nm \]  

Otherwise

\[ T_{cg} = 0.0023447 \omega_c Nm \]
Chapter 8

Load Box

This chapter describes the functioning of the load box and its role in the hybrid power train system. The first section explains how the load box was physically implemented to the system and its control module. Next, follows a section which explains how power can be supplied to the battery while driving “downhill”. The last section of this chapter includes the computer simulation of the load box and how the load affects the equation of torque of the carrier/wheels.

Section 8.01 Load Box Control Module

The load box is not part of the hybrid power train system itself, but is used to simulate the up and down grades of a road. It has a motor/generator (Model: MY6812 Unite Motor) and two power resistors in parallel 0.29Ω. When simulating downgrades the motor/generator acts as a motor in order to drive the output shaft. It is powered by a benchtop power supply. When simulating upward grades the motor/generator acts as a generator connected to the resistor in order to apply an electrical load to the output of the HPT. The intensity of both situations is varied using Pulse Width Modulation (PWM).

Figure 8.1 shows the circuit used for the load box. Both scenarios are changed by controlling the relay’s position by sending signals from the microcontroller (Arduino Mega 2560).
A switch is used to connect or disconnect the load box from the system. This switch is controlled manually by the user. Also, the circuit includes two fuses #1 and #2 to protect it.

In the following sections each of the two possible circuit arrangement will be described:

a) **The circuit behaves as a load**

Figure 8.1 depicts the relay’s position for this first scenario. In this case a load is applied to the system using a 0.14 Ω, XX Watts power resistor (the two 0.29Ω resistors in parallel). In order to apply variable loads a PWM signal is sent from the Arduino into FET #3. By varying the length of the pulse, different load values from (0 to 0.14Ω) can be applied to the HPT.

Table 8.1 depicts the signals that should be sent by the Arduino for each case.
As seen in Table 8.1  the circuit works as a load when FET 3 is set to work with PWM with a range from 0 to 255 and FET 2 and 3 are set to 0.

b) **The circuit behaves as a downhill slope**

The circuit behaves as a downhill slope if the relays changes position. At this point the FET#1, also connected to a PWM pin, controls the speed of the electric motor by varying the average of voltage applied.

Table 8.1 depicts the signals that have to be sent from the Arduino to FET 1, 2 and 3 in order to make the circuit works as a slope.

With these two arrangements the system has variable scenarios that resemble a real road.

**Section 8.02 Adding Power to the system**

It must be mentioned that in order to save energy, the wheels of the car can also be used to charge the battery. This scenario occurs when the car is driving downhill and the generator is driven by the wheels instead of the air motor.

---

**Table 8.1: FET’s Control Signals**

<table>
<thead>
<tr>
<th>Field-Effect transistor</th>
<th>Control signal</th>
<th>Range</th>
<th>PWM Range signal</th>
<th>PWM Load</th>
<th>PWM Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>FET #1</td>
<td>Motor (slope)</td>
<td>(0-max speed)</td>
<td>(0-255)</td>
<td>0</td>
<td>(0-255)</td>
</tr>
<tr>
<td>FET #2</td>
<td>Relay</td>
<td>0N or OFF</td>
<td>0 or 255</td>
<td>0</td>
<td>255</td>
</tr>
<tr>
<td>FET #3</td>
<td>Load magnitude (hill)</td>
<td>(0-0.14Ω)</td>
<td>(0-255)</td>
<td>(0-255)</td>
<td>0</td>
</tr>
</tbody>
</table>
Section 8.03  Simulation of the Load Box

The simulation of the load box is achieved by generating an external torque and applying it to the wheels/carrier of the system. This is done by generating a pulse for a period of time and adding it to the torque produced by the carrier. It will simulate a hill if the sign of the torque generated by this pulse is opposite from the torque generated by the carrier. On the other hand, it will simulate a slope if the contrary occurs.
Chapter 9

Battery box

The state of charge (SOC) of the battery pack plays an important role in the model of the hybrid powertrain. Depending on its value the system has to decide to operate or not with the electric motor. There are two possibilities for modeling the battery pack. The first uses the open-circuit voltage as a measure of the state of charge, while the second uses “Coulomb-counting” to monitor the flow of electrons into and out of the battery pack. The first possibility is explained for the sake of completeness, but the second is used in the powertrain model, for reasons discussed later. Figure 9.1 depicts the theoretical diagram of the battery

![Battery Diagram](image)

*Figure 9.1: Battery Diagram*

Section 9.01 Open-Circuit Voltage vs. State of Charge

Assuming a linear relationship between open-circuit voltage and state of charge:

\[
SOC = K_s v_{oc} + v_{oc}(0)
\]  (9.1)
where \( v_{oc}(0) \) is the open-circuit voltage of the discharged battery pack. The battery pack has been modeled with the circuit diagram shown in Figure 9.1. Here, \( R_b \) is the internal resistance of the battery, and \( v_{oc} \) is the open-circuit voltage across the battery terminals with no load present. The output voltage produced by the battery across a load is:

\[
v_b = v_{oc} - i_b R_b
\]  

(9.2)

where \( i_b \) is the current flowing out of the battery. The power delivered by the battery is

\[
P_b = i_b \cdot v_b = (v_{oc} - i_b R_b) i_b
\]  

(9.3)

At a given time, power may be flowing into the battery from the generator

\[
P_{in} = T_g \omega_g \eta_g
\]  

(9.4)

where \( \eta_g \) is the efficiency of the generator. In the sign convention adopted earlier, the generator torque is defined as positive if it presents a load to the system; that is, if it is in the opposite direction from \( \omega_g \). If the generator is sending power to the battery pack, then \( P_{in} \) is positive. The battery is also used to drive the electric motor, whose power consumption is:

\[
P_{out} = \frac{T_{EM} \omega_{EM}}{\eta_{EM}}
\]

where \( \eta_{EM} \) is the efficiency of the motor. Here the torque and angular velocity are defined as positive in the same direction, and a positive \( P_{out} \) means that the motor is drawing power from the battery. The net power flow out of the battery is then
\[ P_b = \frac{T_{EM} \omega_{EM}}{\eta_{EM}} - T_g \omega_g \eta_g \]  

(9.5)

The state of charge changes as current flows into or out of the battery pack. The relationship between current flow and state of charge is

\[ S\dot{O}C = \frac{i_b}{C_b} \]  

(9.6)

where \( C_b \) is the capacity of the battery in Amp-hours. Equations (9.2, 9.3 and 9.5) can be used to solve for the battery current:

\[ i_b = i_{sc} \pm \sqrt{i_{sc}^2 - \frac{4}{R_b} \left( \frac{T_{EM} \omega_{EM}}{\eta_{EM}} - T_g \omega_g \eta_g \right)} \]  

(9.7)

where:

\[ i_{sc} = \frac{\nu_{oc}}{R_b} \]

is the short circuit current of the battery. The equation of state for the battery pack is then:

\[ S\dot{O}C = \frac{1}{C_{bat}} \left[ i_{sc} \pm \sqrt{i_{sc}^2 - \frac{4}{R_{bat}} \left( \frac{T_{EM} \omega_{EM}}{\eta_{EM}} - T_g \omega_g \eta_g \right)} \right] \]  

(9.8)

To implement this equation, there are several parameters that need to be measured for the battery pack, including the short-circuit current and the internal resistance. Since the internal resistance varies with state of charge, this battery model is rather
complicated, and is strongly affected by the type of battery used. Therefore, the “Coulomb-counting” is chosen to assess the state of charge.

Section 9.02 State of Charge using Coulomb-Counting

If the hybrid powertrain starts with a completely discharged battery, the SOC can be assessed by simply counting the number of electrons entering or leaving the battery during operation. The equation of state for this method is

\[ S\dot{O}C = \frac{i_b}{C_b} \]  \hspace{1cm} (9.9)

We may use simple integration to find the current state of charge

\[ SOC = \frac{1}{C_b} \int_{0}^{t} i_b \, dt \] \hspace{1cm} (9.10)

If the battery starts time \( t_0 \) with a known state of charge, we may use the following equation instead.

\[ SOC = \frac{1}{C_b} \int_{t_0}^{t} i_b \, dt + SOC_0 \] \hspace{1cm} (9.11)
Section 9.03  Generator and Battery Box Control Module: Hardware Implementation

a) Batteries

The system has two 12V, 1.4 Ah rechargeable sealed lead acid batteries (PS-1212 Power Sonic) connected in series.

Figure 9.2: Current Sensor (ACS711EX -15.5A to +15.5A)

b) Current Sensor

As mentioned previously, the current flowing in and out of the battery is monitored in order to know the state of charge. A bidirectional current sensor (ACS711EX -15.5A to +15.5A) is used to measure the current that flows into and out of the battery. Figure 9.2 depicts the current sensor used for the project.

The wiring of this sensor is simple. Figure 9.3 depicts the connections of the sensor. It must have a supply voltage $V_{CC}$ (3V to 5V) to function.

Figure 9.3: Current Sensor Connections
In order to calculate the current, the manufacturer provides the relationship, between the instantaneous input current $i$, and the sensor output voltage, $V_{OUT}$ Equation (9.11). This last signal is the one that is sent to the Arduino. [33]

$$i = 36.7 \left( \frac{V_{OUT}}{V_{cc}} \right) - 18.3 \ A$$

(9.12)

c) **Temperature Sensor**

Inside the battery box is included a temperature sensor and a voltage sensor in order to detect malfunctions of the battery.

The temperature sensor (TMP36) has been added to the battery in order to monitor its operating temperature.

*Figure 9.4: Temperature sensor (TMP36)*

d) **Voltage Sensor**.

The voltage sensor is simply a voltage divider as shown in Figure 9.5
The values of $R_1$ and $R_2$ were calculated using the following equation:

\[ V_{OUT} = \frac{R_2}{R_1 + R_2} V_{IN} \]  \hfill (9.13)

With the aim of protecting the Arduino, $V_{OUT}$ needs to be less or equal to 5V. $V_{IN}$ is 24V. $R_1$ and $R_2$ were selected to be $R_1 = 4.3 \text{ k}\Omega$ and $R_2 = 1 \text{ k}\Omega$. If $V_{IN}$ equals 24V. Substituting the values in Equation (9.13) gives that $V_{OUT} = 4.53$ V. This means that the signal can be read by the Arduino.

*Figure 9.5: Voltage Divider*
Chapter 10

Hardware Design for the Control of the Hybrid Power-train System

This section provides a detailed explanation of the hardware implementation for the control system. These include the two tachometers, the air flow control hardware for the air motor and the electric motor controller.

Section 10.01 Sensing the speed of the motors

The speed of the motors are measured using a student-built tachometer. Figure 10.1 depicts one of the tachometers used for the hybrid power train; it is comprised of a Hall-effect sensor and a rotary sensor wheel commonly known as a “daisy wheel”. Each of the two tachometers is installed in the shaft of the air and the electric motor respectively.

Figure 10.1: Hall Effect Tachometer
Figure 10.2: Schematic Diagram of the Air Flow Control System

(a) **Hardware.** The air motor is powered by compressed air. By controlling the flow of air, the speed of the motor can be varied. Shop air at 120psi (8.3bar) is supplied to an appropriately-sized orifices in an aluminum block. These orifices choke the air depending on their cross sectional areas. The exhaust of each orifice is directed into a set of six solenoid valves (McMaster-Carr number 4711K512). Finally, the air from the valves is combined and sent to the air motor. By opening and closing each solenoid valve, the speed of the air motor can be regulated. A schematic diagram of the system is shown in Figure 10.2.

The air supply is connected to an aluminum block with varying holes sizes; the flow through each hole is proportional to its cross-sectional area. The area of each hole is determined such that the six valves can work simultaneously in a “binary” pattern. That
is, opening the smallest orifice gives the lowest speed (speed “000001”), opening the second smallest gives the second lowest speed (speed “000010”), opening the two smallest simultaneously gives the third speed (speed “000011”) and so on, for a total of 63 different “steps”.

For example in order to activate the step 19 the valves that need to be activated are $A_0$, $A_1$ and $A_4$. This can be seen in Table 10.1.

Table 10.1: Binary Pattern

<table>
<thead>
<tr>
<th>Drill bit</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary pattern</td>
<td>$2^0$</td>
<td>$2^1$</td>
<td>$2^2$</td>
<td>$2^3$</td>
<td>$2^4$</td>
<td>$2^5$</td>
</tr>
<tr>
<td>Sum</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>$A_0+A_1+A_2+A_3+A_4+A_5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example:
Step 19
Binary pattern
<table>
<thead>
<tr>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Sum
| 1     | 2     | 0     | 0     | 16    |

19

For the design of the aluminum block the two variables that are important are the pressure and the flow rate of the supply line, which are 120psig and (0-3.5) CFM.

Selecting the diameter of the orifices

To begin, the maximum air flow available to the system is determined by the inlet fitting of the block which has an ID of 5/35’’ Hex (See Figure 10.3). It must be mentioned that the ID was selected instead of the ID of the 0.25’’ supply line tubing which is 0.18” since it is the limiting factor in the total air flow. Figure 10.3 depicts the type of fitting used for the aluminum block (McMaster-Carr number 5779K108).
When all valves are open, the total cross-sectional area of the orifices must equal the cross-sectional area of the inlet fitting.  

\[ A_{\text{supply}} = A_0 + 2A_0 + 4A_0 + 8A_0 + 16A_0 + 32A_0 \]

where \( A_{\text{supply}} \) is the cross-sectional area of the inlet fitting and \( A_0 \) is the cross-sectional area of the smallest orifice. For this case the \( A_{\text{supply}} \) was calculated as though the area was a circle, using as the diameter 5/32”\). This gives that the \( A_{\text{supply}} \) is 0.01917 in\(^2\). Rearranging this equation to solve for \( A_0 \) gives

\[ A_0 = \frac{A_{\text{supply}}}{63} \]

Of course, the orifice sizes must correspond to existing, standard drill bit sizes. The table below lists the theoretical and actual sizes used in the orifice block.
Table 10.2: Orifice Sizes Flow Control Module #1

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Theoretical Area (in²)</th>
<th>Theoretical Diameter (in)</th>
<th>Drill Bit #</th>
<th>Drill Bit Diameter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 = A₀</td>
<td>3.043E-04</td>
<td>0.01968321</td>
<td>76</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>6.086E-04</td>
<td>0.02783626</td>
<td>70</td>
<td>0.028</td>
</tr>
<tr>
<td>3</td>
<td>1.217E-03</td>
<td>0.03936641</td>
<td>61</td>
<td>0.039</td>
</tr>
<tr>
<td>4</td>
<td>2.434E-03</td>
<td>0.05567251</td>
<td>53</td>
<td>0.0595</td>
</tr>
<tr>
<td>5</td>
<td>4.869E-03</td>
<td>0.07873282</td>
<td>47</td>
<td>0.0785</td>
</tr>
<tr>
<td>6</td>
<td>9.737E-03</td>
<td>0.11134503</td>
<td>34</td>
<td>0.111</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>0.33263624</td>
<td></td>
<td>0.336</td>
</tr>
</tbody>
</table>

\[ A_{supply} = 0.01917 \]

After testing the block it was noted that there were losses in the flow through the orifices of the block (e.g., friction). In consequence, the maximum speed is not the same as if the air motor was connected directly to the air line. Also, it was observed that at some point the speed of the motor was not increasing even though the total cross-sectional area of the orifices was increasing. Using this orifice block the motor reaches its maximum at the step 24, when the area equals 0.006964 in². That means that the design of the block is limited by the pressure of the line which is approximately 120 psi.

In order to give more resolution to the number of steps a smaller bit was chosen #80 as A₀. Table 2 shows the new values chosen for the holes areas.

Table 10.3: Orifice Sizes Flow Control Module #2

<table>
<thead>
<tr>
<th>Orifice</th>
<th>Theoretical Area (in²)</th>
<th>Diameter (in)</th>
<th>Drill Bit #</th>
<th>Drill Bit Diameter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 = A₀</td>
<td>1.43E-04</td>
<td>0.0135</td>
<td>80</td>
<td>0.0135</td>
</tr>
<tr>
<td>2</td>
<td>2.86E-04</td>
<td>0.01909188</td>
<td>76</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>5.73E-04</td>
<td>0.027</td>
<td>71</td>
<td>0.026</td>
</tr>
<tr>
<td>4</td>
<td>1.15E-03</td>
<td>0.03818377</td>
<td>62</td>
<td>0.038</td>
</tr>
<tr>
<td>5</td>
<td>2.29E-03</td>
<td>0.054</td>
<td>54</td>
<td>0.055</td>
</tr>
<tr>
<td>6</td>
<td>4.58E-03</td>
<td>0.07636753</td>
<td>48</td>
<td>0.076</td>
</tr>
</tbody>
</table>

\[ A_{supply} = 9.02E-03 \]

\[ A_{supply} = 0.22814318 \]

\[ A_{supply} = 0.2285 \]
The $A_{\text{supply}}$ in this case is 0.00901775 in$^2$. It must be mentioned that the reason why the $A_{\text{supply}}$ is much smaller than the ID of the fittings is because the system has been limited by the line of the shop. In this case, the main purpose is to start with the smallest bit available. After testing the design, it was noted that the air flow did not increase after step 49. The area at this step is 0.006912 in$^2$. That means that at this point the system reaches its maximum. Even though the system cannot take advantage of the 63 steps available, the second block provides a higher resolution at no load in comparison to the first block that provided only 24 steps before it reached the maximum. That is the reason the second block was selected for the project.

(b) Speed Control System

The Arduino controls the six solenoid valves using the “binary” pattern describes in section (a). The solenoid valves are connected to six digital outputs of the Arduino (ON/OFF). Depending requested speed of the air motor the microcontroller sends the required signals to activate the desired combination of valves.

Section 10.03 Current Control Module

(a) The Hardware

The electric motor is powered by a two 12 V lead acid batteries connected in series (PS-1212 12V 1.4Ah). The actuator is a motor driver (Motor Driver 15A IRF7862PBF) which regulates the current depending on the signals it receives from the microcontroller. The motor controller is basically an “H-Bridge”, that allows the motor move forward or backward at different speeds.
(b) **Speed Control System.** The motor controller receives two signals from the Arduino. The first one is a digital signal. Depending on it the electric motor can move forward or backward. The second signal is an analog signal PWM (Pulse Width Modulation), which allows the motor to reach different speeds. The PWM signal ranges from 0 to 255, with 0 being the minimum value for the signal and 255 the maximum. Taking both signals into consideration the range of the action signal for the code is between -255 to 255 with positive numbers corresponding to forward speeds and negative numbers corresponding to reverse speeds.
Chapter 11

Control Module of the Hybrid Power-train System

Chapter 11 includes the programming of the variable transmission. Specifically, the decision making algorithm of the system. It also contains, the PID controller for each of the motors in order to achieve the cruise control system. As well as, the tracking of the SOC (State of Charge) of the battery, some safety features and finally the load box values for the code.

Section 11.01 Control Module of the Hybrid Electric Power-Train

As mentioned in other occasions, the main purpose of the programming is to implement a cruise control system. That means that no matter the load applied, the velocities of both the air and electric motor should adjust themselves automatically to achieve the desire output speed.

The system is controlled by an Arduino Mega 2560. The microcontroller is programmed using the Arduino programming language (based on Wiring). This microcontroller monitors and makes the decisions for the system. Figure 11.1 shows the

Figure 11.1: Microcontroller Arduino Mega 2560
Arduino Mega 2560.

Thanks to the microcontroller, sensors and actuators, the system can behave as a continuously variable transmission. Thanks to this variable transmission the system achieves excellent responses and takes advantage of the motor operations efficiency ranges. The system determines the desired operating condition and the existing operating condition, and then it controls the motors, generator, battery, and other components in real time to achieve the desired output.

An outline of the control of the HPTS can be seen in Figure 11.2. The Arduino has control over the speed of both the air and electric motor. Also, it monitors the battery state of charge (SOC) and it can connect/disconnect the generator to/from the system. Finally it controls the load applied to the system in order to simulate hills or slopes as a common road. Additional, for safety the system has sensors to monitor the temperature and voltage of the battery to detect malfunctions. In that case the Arduino sends a signal and turns the electric motor off.

![Control Schematic](image_url)

*Figure 11.2: Control Schematic*
Finally, to achieve the cruise control system implementation, two independent proportional-integral-derivative controllers (PID controller) are integrated in the code. One for each motor, since they are the power source engaged to the driving wheels of the car.

**Section 11.02 Decision Making Algorithm**

Depending on the value of certain variables the microcontroller makes the require decisions to vary the transmission in order to achieve maximum performance. The main decisions of the system depend of these three variables:

a) The set value, magnitude of the output/driving shaft (set by the user).

b) The state of charge of the battery

c) The output speed

a) **The set value, magnitude of the output (set by the user).**

For the first case, the set value of the output plays an important role in the decision making. Depending on this set value the system will require to use only the electric motor or both motors simultaneously.

1. **The set value is slow (<32 rad/s).**
Noting that the efficiency of the air engine at slow speeds is very low (see Appendix 1). The electric motor is the responsible of providing the motion of the system in this scenario. After analyzing the efficiency tables of the air engine, the value that is considered as slow was to selected as 32 rad/s. At this point it can be noticed that the efficiency of the air engine is about 1.5% or less which is considered as poor.

2. **The set value is fast (≥32 rad/s).**
In this case, the air engine is set to run at its optimal speed; that means where the efficiency of the air engine is highest. On the other hand the electric motor set value adjusts itself to provide the difference to reach the desire output set value. The optimal
speed of the air engine according to the appendix 1 is between 84 rad/sec and 103 rad/sec. For this project 110 rad/sec was selected as the optimal speed.

b) **The state of charge of the battery**

The second important value that has to be monitored is the battery state of charge. The state of charge of the battery plays an important role for the decision making of the system, since the electric motor depends on its power. Also depending on the SOC the generator can be disconnected from the system. For the battery state of charge three cases can be found:

1. **There battery is full.**

   In this case the generator is disconnected from the system. Also, the decision making of the system are the same mentioned for case (a).

2. **There is no battery.**

   Since the electric motor cannot operate, the air engine is responsible of generating the mechanical motion to maintain the required constant speed of the system regardless of the magnitude of the set value. That means that not necessarily the air engine will go at its optimal speed.

3. **Driving under normal conditions and the battery SOC is less than 30%**

   In this case both motors run simultaneously and the generator in connected to the system.

c) **The Output/Carrier speed**

The third variable that affects the decision making is the output speed. If the output speed is greater than certain value then the system can connect the generator to charge the battery. In this case this value is selected as 274.7882 rad/sec. This value was calculated is Chapter 7, Equation (7.6) resulting in Equation (7.13). Just for a reminder, at this speed is when the voltage generated is high enough to start charging the battery.

With these variables in mind, three cases of operation can be described:

**Case 1. Air engine (AM) works by itself:** Not necessarily at its optimal speed.
This occurs when there is no battery. The powertrain behaves as a regular transmission.

**Case 2. Electric Motor works by itself:** This occurs when the set value is set to a slow speed. In the case of this project when it is less than 32rad/sec.

**Case 3. Both motors work simultaneously:** This occurs when there is battery and the set speed of the wheels is greater than 32rad/sec. For this case the AM operates at its optimal speed and the electric motor compensates to reach the desired set value.

After the microcontroller decides with what motor is going to operate, the cruise control system is achieved by including two PID controllers to the code.

Depending on the current speed, the system accelerates or decelerates on its own to achieve the desired set value. Each motor has an independent PID controller that will be discussed in detail in section 11.04.

<table>
<thead>
<tr>
<th>Table 11.1: Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SOC ≥30%</strong></td>
</tr>
<tr>
<td>Set Value &lt; 32 rad/s</td>
</tr>
<tr>
<td>Set Value ≥ 32 rad/s</td>
</tr>
<tr>
<td><strong>SOC &lt; 30%</strong></td>
</tr>
<tr>
<td>Set Value &lt; 32 rad/s</td>
</tr>
<tr>
<td>Set Value ≥ 32 rad/s</td>
</tr>
</tbody>
</table>

| Generator |
|-----------------
| Output/Carrier ≥ 275 rad/s & SOC<99% | Connect |
| Output/Carrier < 275 rad/s | Disconnect |
Figure 11.3 depicts a general control flowchart to have a variable transmission.

**Figure 11.3: Hybrid Power Train Algorithm**

1. **Set Value Output/carrier**
   - **SOC >30%**
   - **Case 1: AE works by itself. Not necessary at its optimal speed.**
   - **Case 2: EM motor works by itself**
   - **Case 3: AE works at its optimal speed. EM compensates to reach the desire set value.**

2. **Reading from sensors:**
   - **Tachometer:** Rotational Speed $\omega_c$
   - **$\omega_c > 1000$ RPM**
   - **NO**
   - **YES**
   - **Connect Generator**
   - **Increase the steps of operating motor(s)**
   - **Decrease the steps of the operating motor(s)**

3. **PID controller**
   - **Is the $\omega_c$ > Set Value?**
   - **YES**
   - **NO**
Section 11.03 Set Value: Air and Electric Motors

The *Set Value* of the driving gear is the speed that the user wants to reach and maintain with the *Cruise Control System*. Depending of the input values sent to the Arduino, the microcontroller decides with which motor to operate. When this decision is made the set value of each motor is calculated with Equation (11.1) which describes the behavior of the planetary gears. This equation is used to calculate the set values that each of the motors has to reach as followed:

\[
setvalue_c = \frac{setvalue_{AM} + setvalue_{EM}}{2} \quad (11.1)
\]

Depending of each of the cases explained in the previous sections the motors operate either: simultaneously or solo. The gear ratio between the power shafts and the planetary gears has been considered when calculating the actual RPM of the system.

**Case 1. AM operates by itself**

For the first case when there is no battery, obviously the speed of the EM is equal to cero since there is no power. Equation (11.1) is solved to find the

\[
setvalue_{AM}.
\]

\[
setvalue_{AM} = 2 \; setvalue_c - setvalue_{EM} \quad (11.2)
\]

\[
setvalue_{EM} = 0 \quad (11.3)
\]

\[
setvalue_{AM} = 2 \; setvalue_c \quad (11.4)
\]

**Case 2. EM operates by itself**

For slow speed the electric motor has to operate alone. Solving Equation (11.1) for the EM gives:

(11.1) for the EM gives:
\[ \text{setvalue}_{EM} = 2 \text{setvalue}_C - \text{setvalue}_AM \]  
\[ \text{setvalue}_AM = 0 \]  
\[ \text{setvalue}_{EM} = 2 \text{setvalue}_C \]  

Case 3. Both motors operate simultaneously

As mentioned, for normal driving conditions the air engine operates at its optimal speed. In consequence the \( \text{set value}_AM \) is constant. On the other hand, the electric motor compensates to reach the desire speed.

\[ \text{setvalue}_AM = \text{optimal speed} = 1000 \text{ rpm} \]  
\[ \text{setvalue}_{EM} = 2 \text{setvalue}_C - 1000 \]

Section 11.04 PID controller

The PID controller is named after its first letters of the names of the individual terms. These are Proportional, Integral and Derivative terms in the controller. The PID controllers are widely use in the industry. For this project a PID controller is included for the two input forces of the system in order to implement a cruise control system.

a) PID controller: Air Engine

A PID controller is used regulate the speed of the motors under varying loads. The PID controller formula implemented in the microcontroller code is:

\[ U_1 = U_0 + K_{PAM} e + K_{IAM} \int_0^t e \, dt + K_{dam} \frac{de}{dt} \]  

87
Where $U1$ is the control action or New Step, which ranges from 0 to 63, $U0$ is the previous step or old step, and $KpAM$, $KiAM$ and $KdAM$ are the proportional, integral and derivative gain respectively for the air engine. Furthermore, $e$ is the error and is calculated by the difference between the air engine’s set value and the actual speed.

$$error = set\ value_{AM} - \omega_{AM}$$

(11.11)

The controller gains for the air engine were found experimentally using the manually tuning parameters.

<table>
<thead>
<tr>
<th>$K_{pAM}$</th>
<th>$K_{iAM}$</th>
<th>$K_{dAM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>0.0001</td>
<td>0.000001</td>
</tr>
</tbody>
</table>

Figure 11.4 depict with detail the algorithm used for control the air engine. As seen in Figure 11.3, the air engine operates in case 1 and 3. These cases are presented below.
Case 1: EM no battery.
· AE works alone. Not necessary at its optimal speed. The set value AE varies.
  · $W_{EM}=0$.

Case 3:
· AE works at its optimal speed: Set Value AE (constant).
  · EM compensates to reach the desired output set value.

$$setvalue_{AE} = \text{optimal speed}$$

$$error = setvalue_{AE} - \omega_{AE}$$

$$\text{New Step} = \text{Old Step} + K_{p_{AE}} \cdot error + K_{i_{AE}} \cdot \int (0-t)error \cdot dt + K_{d_{AE}} \cdot derror/dt$$

Action:
The required solenoid valves are activated.
Range Steps = [0-63]
b) **PID controller: Electric Motor**

As mentioned in previous section the electric motor is also controlled by a PID controller. The PID controller formula is similar to Equation (11.10) but applied for the electric motor.

\[
U_1 = U_0 + K_{pEM} e + K_{iEM} \int e \, dt + K_{dEM} \frac{de}{dt} \quad (11.12)
\]

Where $U_1$ is the control action or *New Step*, $U_0$ is the previous step or *old step*. For the electric motor these values range is between -255 to 255. It can be noticed that equation is very similar. What differ are the values of the gains and as mentioned the velocity range.

The error is calculated as followed:

\[
error = set \, value_{EM} - \omega_{EM} \quad (11.13)
\]

The controller gains were found experimentally using the manually tuning parameters.

<table>
<thead>
<tr>
<th>$K_{pEM}$</th>
<th>$K_{iEM}$</th>
<th>$K_{dEM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The electric motor control flowchart is the following:
Figure 11.5: Electric Motor Algorithm
Section 11.05 Note PID controller

In order to make the code simpler a PID Arduino library was added to the code. This library includes Equation (11.11) and (11.10). But also other details to make the PID more reliable and robust since it takes into consideration other variables such as: the sample time, the error every time you change the set value or even if you want to change the gains during the run, also turning ON or OFF the PIDs, among others. It was developed by Brett Beauregard. For more detail information about the PID library visit: http://brettbeauregard.com/blog/2012/01/arduino-pid-autotune-library/

Section 11.06 First Estimation of the Require Velocity

A more rapid stabilization period can be achieved for each of the motor every time that the set value changes. This is done by giving a first estimate of the step that has to be taken (AM [0 to 63] or EM [-255 to 255]) in order to reach the desire speed.

The no-load speed was measured at each of the paces for each motor. After plotting all the different points, equations were found for each of the motors.

![Figure 11.6: Inverse of the electric motor behavior](image)

\[ U_c = 0.1038 \times \text{set value EM} + 3.5965 \]

\[ R^2 = 0.9996 \]
The approximation of the first step [-255 to 255] which the electric motor should operate is calculated as followed:

\[ U_{0EM} = 0.1038 \times (set \ value_{EM}) + 3.5965 \]  

Where \( U_{0EM} \) is the step of the electric motor. This equation plays an important role in the control system since it allows the controller to make a first estimation of the pace on which the electric motor must operate. As mentioned in chapter 5 the paces of the electric motor rages between [-255 and 255]. These paces control the velocity and direction of the electric motor.

The same procedure was done with the air engine. Figure 11.7 represents the behavior of the motor at no load. Figure 11.8 is the same data but the inverse in order to find the required step value depending on the RPM of the set value.

\[ y = 902.9 \ln(x) - 633.94 \]

\[ R^2 = 0.9579 \]

Figure 11.7: Air Engine Behavior Step by Step
The following Equation was found:

\[ U_{0AM} = 2.1894^{0.0011 \times setval_{AM}} \]  \hspace{1cm} (11.15)

The Equation found for this case was not linear since the system is limited by the line pressure and the speed of the motor cannot increase after a certain step as mentioned in Chapter 10.

This procedure is just to give a first estimated in the control algorithm. Since the load is not taken into account it will only be accurate when there is no load applied to the system, however, after this step a PID controller is used for fine tuning the speed.

**Section 11.07 Battery**

a) **State of charge**

The battery SOC calculation was explained in Chapter 9 using Equation (9.11). For the code the variable of this equation will be calculated as followed:
• Current

A bidirectional current sensor (ACS711EX -15.5A to +15.5A) it is used to measure the current that flow in and out the battery.

In order to calculate the current, the manufacturer provides the relationship between the instantaneous input current $i$, and the sensor output voltage, $V_{OUT}$. This last signal is the one that is sent to the arduino. In this Equation $V_{CC}$ is the supply voltage (range 3 V to 5 V). (add reference)

$$i = 36.7 \frac{V_{OUT}}{V_{cc}} - 18.3 \, A$$

• Time

The delta time is measured by the Arduino.

$$\Delta t = t_f - t_0$$

• SOC

For programming purposes the SOC is calculated as followed:

$$SOC = \frac{i_{bat} \ast \Delta t}{C_{bat}} + SOC_0$$

Where $C_{bat}$ is the capacity of the battery that is 1.4 Ah

After the reading the value of the SOC is stored as $SOC_0$. It must be noticed that the $SOC_0$ of the battery should be stored even though the system is reset or shut off. Otherwise, the program will reset the state of charge and the next calculation of the SOC will not be correct. In order to achieve this, an Electrically Erasable Programmable Read-Only Memory (EEPROM) is used. This is a type of non-volatile memory used in computers and other electronic devices to store small amounts of data that must be saved
when power is removed, e.g., calibration tables or device configuration. The Arduino Mega 2560 includes this memory.

b) **Safety Feature**

Every project should include safety features. In this case a temperature sensor (TMP36) has been added to the battery in order to monitors its operating temperature.

The sensor is connected to an analog input to the Arduino and it is programmed to give the temperature in Celsius and Fahrenheit.

This sensor is added for operational safety. If the system senses high temperature in the battery, the electric motor will be automatically turned off. On the other hand, a visual signal will be also added: a red LED will turn on. This way the user can detect that something is wrong with the system.

**Section 11.08 Load Box**

The Load is used in order to simulated hills or slopes as a common road. Chapter 8 gives a detail explanation of its functionality. A manual switch has been programmed in order to be set the uphill or downhill mode. The intensity of the load at each state is also manually control by a potentiometer.

**Table 11.2: Signals Arduino**

<table>
<thead>
<tr>
<th>FET-Effect transistor</th>
<th>Control signal</th>
<th>Range</th>
<th>PWM Load</th>
<th>PWM Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>FET #1</td>
<td>Motor (slope)</td>
<td>(0-max speed)</td>
<td>255</td>
<td>(0-255)</td>
</tr>
<tr>
<td>FET #2</td>
<td>Relay 0N or OFF</td>
<td>0</td>
<td>255</td>
<td>0</td>
</tr>
<tr>
<td>FET #3</td>
<td>Load magnitude (hill)</td>
<td>(0- XXΩ)</td>
<td>(0-255)</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

96
Chapter 12

Implementing the Hybrid Power Train in a Classroom Setting

This chapter contains information about the implementation of the HPT. It can be used by instructors at other institutions. More specifically, it states the components that should be provided to the students and the ones that they must build, install and program. Also, it summarizes the implementation of the modules for the hybrid-electric powertrain prototype.

Section 12.01 Providing the students with a Station

The students will design and fabricate the main components within the HPT: the air motor, the planetary gears and the tachometers. The rest of the components will be provided to the students in a station: the battery pack, load box, electric motor, generator, solenoid valves, microcontroller and motor driver. The station is shared by 4 groups. A summary of these components is provided below.

**Battery Pack:** two 12V batteries (Power Sonic, Model: PS-1212, 12V-1.4Ah) that powers the 24V electric motor (Unite Motor, Model: MY6812). The Battery Pack also includes temperature (Analog Devices, Model:TMP36), current (Polulu, ACS711EX -15.5A to +15.5A) and voltage (voltage divider) sensors to monitor the state of charge of the battery. The students are required to write code to interpret the sensor readings and respond appropriately.

**Load Box:** the load box is used to simulate uphills and downhill on a road. It has a motor/generator (Model: MY6812 Unite Motor) and two power resistors in parallel 0.29Ω. When simulating down grades the motor/generator acts as a motor in order to
drive the output shaft. It is powered by a benchtop power supply. When simulating upward grades the motor/generator acts as a generator connected to the resistor in order to apply an electrical load to the output of the HPT. The intensity of both situations is varied using Pulse Width Modulation (PWM).

**Electric Motor and Generator:** both are provided to students. In the case of the electric motor, a motor controller (Polulu, Motor Driver 15A IRF7862PBF) is also provided in order to control the speed and direction of the motor using the Arduino.

**Solenoid Valves:** six solenoid valves (McMaster-Carr number 4711K512) are used to regulate the flow of air with the aim of controlling the speed of the air motor. The students will design and fabricate an orifice block to regulate the amount of air flowing through each solenoid, as described below.

**Motor Controller:** a motor driver (15A IRF7862PBF) is provided in order to control the electric motor.

**Arduino UNO:** each group is provided with a microcontroller Arduino UNO in order to make the programming of the variable transmission.

**Section 12.02 Successfully Implemented Modules**

All of the modules have been successfully implemented except for the “Overall Control System”, that is being held at the present semester (Spring 2014). The design/build project is provided below.
Module I: Tachometer

The consists of a Hall-effect sensor and a rotary sensor wheel commonly known as a “daisy wheel”. Each tachometer is installed on the shaft of the air engine and the electric motor respectively. This module is relatively simple, and required students to conduct simple circuit design and fabrication. A few critical concepts are presented, including simple data acquisition, current limiting, and basic transistor theory. This module has been successfully completed by two sets of ME sophomores.

Module II: Air Engine

During their Thermal-Fluid Sciences I course, the Junior ME students are required to build an engine powered by shop air. The learning outcomes for the project are as follows:

1. Design and fabricate a functioning air-powered reciprocating engine.

Table 12.1: Design/Build Project Timeline

<table>
<thead>
<tr>
<th>Year</th>
<th>Semester</th>
<th>Course</th>
<th>Module</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2011-2012)</td>
<td>Fall</td>
<td>-</td>
<td>Arduino-based tachometer</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>ME Laboratory</td>
<td></td>
</tr>
<tr>
<td>2 (2012-2013)</td>
<td>Fall</td>
<td>Thermal-Fluid Sciences I</td>
<td>Air-powered motor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Machine Design</td>
<td>Planetary gearset</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Spring Thermal-Fluid Sciences II</td>
<td>Air valve selection and implementation</td>
</tr>
<tr>
<td>3 (2013-2014)</td>
<td>Fall</td>
<td>System Dynamics and Control I</td>
<td>Electric motor speed control</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>System Dynamics and Control II</td>
<td>Overall control system</td>
</tr>
</tbody>
</table>
2. Use Thermodynamic and Mechanical Design principles to maximize the efficiency of the engine. This is accomplished through optimization of cylinder bore, stroke length, valve timing and other design variables.

Several classes of Rowan University students have successfully completed the air engine project. The students are also required to obtain the efficiency curves of their engine. Paper [34] gives detailed information of how this module was implemented.

Module III: Planetary Gearset

The type of planetary gearset built for the prototype was a differential gearset, but the students are free to choose their own gear topologies. Figure 12.1 shows the planetary gears used for the system. The distinguishing feature of the differential is its reliance upon miter gears rather than spur or helical gears. In the professor’s experience, high-quality miter gears are much easier to obtain at low cost than equivalent spur or helical gears; thus, the selection of a differential gearset.

Figure 12.1: Planetary Gearset used for the HPT prototype
The faculty also built prototypes of the more customary sun/ring/planet and two
suns/two planets gearsets (see rset built by one student team.

Figure 12.2) but these were found to be less smooth in operation, owing to the
inferior quality of the inexpensive spur gears. The students were allowed to choose any
topology for their planetary gearsets, and most chose the sun/ring/planet or differential
configurations.

Figure 12.3 shows the planetary gearset built by one student team.

Figure 12.2: Other planetary gearsets to serve as examples for students.

On the left is a two-sun/two planet configuration and on the right is the traditional
sun/ring/planet configuration.
Module IV: Air valve selection and implementation

The valves are provided but the orifice block is designed and built by the students. Figure 12.4 depicts the solenoid valves and aluminum orifice block used for the prototype.
Module V: Electric motor speed control

During the fall semester of their senior year, students design and implement a strategy for controlling the speed of one of the DC electric motors in the HPT. This module is completed in their System Dynamics and Control I course. Each group was provided with an electric motor, a motor controller and an Arduino UNO. To implement their control strategy, the students create a simple Proportional, Integral and Derivative (PID) controller in Arduino, with speed measured using the tachometers built earlier. The motors must maintain the setpoint despite variations in load, which is provided by the load box described earlier.

Module VI: Overall control system

This module is completed in the System Dynamics and Control II course, during the last semester of the students’ senior year. The students must integrate the whole system with the aim of completing their own hybrid powertrain. Each student group is
assigned to a station (although each station is shared by four student groups). Each group
uses its own air engine, planetary gearset, tachometers and air flow orifice block. The
students must program the Arduino in order to control and monitor all the components of
the system.

Final integration requires the students to design and test an appropriate system-
wide control strategy. Possible student competitions will include tests to see how “far”
each team’s powertrain can travel through a variable drive cycle with a fixed amount of
compressed air available. The main learning outcome for this module is the design and
implementation of a digital controller. This outcome will be assessed through observing
the effectiveness of the student controller designs and also the efficiency of their overall
hybrid powertrain systems.
Chapter 13

Conclusions

A small scale hybrid power train was developed and implemented in an educational environment. The system includes: planetary gear, air engine, electric motor, generator, load box and battery box. This was done using the equipment of the machine shop at Rowan University such as a water jet, milling machine, and lathe, among others.

A variable transmission similar to the Toyota Hybrid System (THS) was programmed. Depending on the driving conditions the system operates with both power sources or independently. This was achieved using a microcontroller (Arduino Mega 2560).

The conditions for the programming are shown in the following table.

<table>
<thead>
<tr>
<th>Table 13.1: Scenarios</th>
<th>Electric Motor</th>
<th>Air Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SOC ≥30%</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set Value &lt; 32 rad/s</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>Set Value ≥ 32 rad/s</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>SOC &lt; 30%</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set Value &lt; 32 rad/s</td>
<td>X</td>
<td>✓</td>
</tr>
<tr>
<td>Set Value ≥ 32 rad/s</td>
<td>X</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Generator</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output/Carrier ≥ 275 rad/s &amp; SOC&lt;99%</td>
<td>Connect</td>
<td></td>
</tr>
<tr>
<td>Output/Carrier &lt;275 rad/s</td>
<td>Disconnect</td>
<td></td>
</tr>
</tbody>
</table>

The outcome of the programming was successful since the system was responding to the conditions set in the coding. It must be notice that for future applications the generator should be geared up. That way the output/carrier speed will start charging the battery at a lower speed. At the present time the Output/Carrier speed needs to be greater
than 275 rad/s to start charging the battery. This speed is considered very high for the system.

A cruise control system was implemented using two PID controllers; the first one, for the air engine and the second for the electric motor. The system reaches quick and stable responses.

<table>
<thead>
<tr>
<th>Table 13.2: Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Time for reaching the set value</td>
</tr>
<tr>
<td>Level of Stability</td>
</tr>
</tbody>
</table>

The mathematical behavior of the components of the hybrid power train was found. The main purpose was to give the reader a more extensive explanation of the components. In this case: planetary gears, air engine, electric motor, generator, load box and battery box. The mathematical model was used to simulate the system using Simulink. The PID controller was not included in the simulation.

A great achievement of this thesis was to prove that it is possible to implement a long-term project in a classroom setting; integrating the main courses of the mechanical engineering curriculum. The key of this success was the collaboration of the faculty responsible of teaching each of the modules.
List of References


## Appendix A. Air Engine Efficiency

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque (in-lb)</th>
<th>Torque (N*m)</th>
<th>$\omega$ (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m$^3$/s$^2$)</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>273.19</td>
<td>0.234</td>
<td>0.026</td>
<td>28.61</td>
<td>0.7</td>
<td>3.3033E-04</td>
<td>0.341737%</td>
</tr>
<tr>
<td>10.2</td>
<td>277.17</td>
<td>0.655</td>
<td>0.074</td>
<td>29.02</td>
<td>0.7</td>
<td>3.3033E-04</td>
<td>0.970800%</td>
</tr>
<tr>
<td>10.3</td>
<td>258.38</td>
<td>1.030</td>
<td>0.116</td>
<td>27.06</td>
<td>0.7</td>
<td>3.3033E-04</td>
<td>1.422294%</td>
</tr>
<tr>
<td>10.3</td>
<td>258.16</td>
<td>0.562</td>
<td>0.063</td>
<td>27.03</td>
<td>0.7</td>
<td>3.3033E-04</td>
<td>0.775066%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.1</td>
<td>483.94</td>
<td>0.883</td>
<td>0.100</td>
<td>50.67</td>
<td>1.4</td>
<td>6.6066E-04</td>
<td>1.141414%</td>
</tr>
<tr>
<td>15.2</td>
<td>402.88</td>
<td>1.992</td>
<td>0.225</td>
<td>42.18</td>
<td>1.4</td>
<td>6.6066E-04</td>
<td>2.145176%</td>
</tr>
<tr>
<td>15.3</td>
<td>355.54</td>
<td>2.116</td>
<td>0.239</td>
<td>37.22</td>
<td>1.4</td>
<td>6.6066E-04</td>
<td>2.010741%</td>
</tr>
<tr>
<td>15.4</td>
<td>288.41</td>
<td>1.893</td>
<td>0.214</td>
<td>30.20</td>
<td>1.4</td>
<td>6.6066E-04</td>
<td>1.458951%</td>
</tr>
<tr>
<td>15.5</td>
<td>256.09</td>
<td>1.403</td>
<td>0.158</td>
<td>26.81</td>
<td>1.4</td>
<td>6.6066E-04</td>
<td>0.959896%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.1</td>
<td>751.17</td>
<td>1.712</td>
<td>0.193</td>
<td>78.66</td>
<td>1.95</td>
<td>9.2021E-04</td>
<td>2.467342%</td>
</tr>
<tr>
<td>20.2</td>
<td>606.47</td>
<td>1.849</td>
<td>0.209</td>
<td>63.51</td>
<td>1.95</td>
<td>9.2021E-04</td>
<td>2.151568%</td>
</tr>
<tr>
<td>20.3</td>
<td>547.77</td>
<td>2.668</td>
<td>0.301</td>
<td>57.36</td>
<td>1.95</td>
<td>9.2021E-04</td>
<td>2.804238%</td>
</tr>
<tr>
<td>20.4</td>
<td>459.54</td>
<td>3.081</td>
<td>0.348</td>
<td>48.12</td>
<td>1.95</td>
<td>9.2021E-04</td>
<td>2.717163%</td>
</tr>
<tr>
<td>20.5</td>
<td>328.08</td>
<td>3.025</td>
<td>0.342</td>
<td>34.36</td>
<td>1.95</td>
<td>9.2021E-04</td>
<td>1.904437%</td>
</tr>
</tbody>
</table>
### Appendix A. Air Engine Efficiency (Continued)

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N·m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m³/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.1</td>
<td>918.86</td>
<td>2.422</td>
<td>0.274</td>
<td>96.22</td>
<td>2.5</td>
<td>1.1798E-03</td>
<td>3.331815%</td>
</tr>
<tr>
<td>25.2</td>
<td>741.51</td>
<td>2.092</td>
<td>0.236</td>
<td>77.65</td>
<td>2.5</td>
<td>1.1798E-03</td>
<td>2.322032%</td>
</tr>
<tr>
<td>25.3</td>
<td>615.52</td>
<td>3.370</td>
<td>0.381</td>
<td>64.46</td>
<td>2.5</td>
<td>1.1798E-03</td>
<td>3.104700%</td>
</tr>
<tr>
<td>25.4</td>
<td>446.65</td>
<td>3.803</td>
<td>0.430</td>
<td>46.77</td>
<td>2.5</td>
<td>1.1798E-03</td>
<td>2.542342%</td>
</tr>
<tr>
<td>25.5</td>
<td>320.51</td>
<td>3.426</td>
<td>0.387</td>
<td>33.56</td>
<td>2.5</td>
<td>1.1798E-03</td>
<td>1.643593%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N·m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m³/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.1</td>
<td>961.32</td>
<td>3.092</td>
<td>0.349</td>
<td>100.67</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>3.834936%</td>
</tr>
<tr>
<td>30.2</td>
<td>930.74</td>
<td>3.149</td>
<td>0.356</td>
<td>97.47</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>3.781964%</td>
</tr>
<tr>
<td>30.3</td>
<td>806.05</td>
<td>4.275</td>
<td>0.483</td>
<td>84.41</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>4.446794%</td>
</tr>
<tr>
<td>30.4</td>
<td>652.87</td>
<td>4.850</td>
<td>0.548</td>
<td>68.37</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>4.085789%</td>
</tr>
<tr>
<td>30.5</td>
<td>520.76</td>
<td>3.953</td>
<td>0.447</td>
<td>54.53</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>2.656647%</td>
</tr>
<tr>
<td>30.6</td>
<td>388.52</td>
<td>4.781</td>
<td>0.540</td>
<td>40.69</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>2.396860%</td>
</tr>
<tr>
<td>30.7</td>
<td>288.06</td>
<td>4.919</td>
<td>0.556</td>
<td>30.17</td>
<td>2.9</td>
<td>1.3685E-03</td>
<td>1.828343%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N·m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m³/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.1</td>
<td>973.50</td>
<td>2.191</td>
<td>0.248</td>
<td>101.94</td>
<td>3.25</td>
<td>1.5337E-03</td>
<td>2.456238%</td>
</tr>
<tr>
<td>35.2</td>
<td>838.36</td>
<td>3.724</td>
<td>0.421</td>
<td>87.79</td>
<td>3.25</td>
<td>1.5337E-03</td>
<td>3.594405%</td>
</tr>
</tbody>
</table>
Appendix A. Air Engine Efficiency (Continued)

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N\textperiodcentered m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m\textsuperscript{3}/s)\textsuperscript{2}</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.3</td>
<td>684.91</td>
<td>4.946</td>
<td>0.559</td>
<td>71.72</td>
<td>3.25</td>
<td>1.5337E-03</td>
<td>3.900834%</td>
</tr>
<tr>
<td>35.4</td>
<td>438.90</td>
<td>5.351</td>
<td>0.605</td>
<td>45.96</td>
<td>3.25</td>
<td>1.5337E-03</td>
<td>2.704182%</td>
</tr>
<tr>
<td>35.5</td>
<td>346.77</td>
<td>4.983</td>
<td>0.563</td>
<td>36.31</td>
<td>3.25</td>
<td>1.5337E-03</td>
<td>1.989655%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N\textperiodcentered m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m\textsuperscript{3}/s)\textsuperscript{2}</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.1</td>
<td>1161.10</td>
<td>3.034</td>
<td>0.343</td>
<td>121.59</td>
<td>3.35</td>
<td>1.5809E-03</td>
<td>3.935164%</td>
</tr>
<tr>
<td>40.2</td>
<td>869.06</td>
<td>3.735</td>
<td>0.422</td>
<td>91.01</td>
<td>3.35</td>
<td>1.5809E-03</td>
<td>3.625985%</td>
</tr>
<tr>
<td>40.3</td>
<td>680.35</td>
<td>4.551</td>
<td>0.514</td>
<td>71.25</td>
<td>3.35</td>
<td>1.5809E-03</td>
<td>3.458749%</td>
</tr>
<tr>
<td>40.4</td>
<td>571.84</td>
<td>5.539</td>
<td>0.626</td>
<td>59.88</td>
<td>3.35</td>
<td>1.5809E-03</td>
<td>3.538442%</td>
</tr>
<tr>
<td>40.5</td>
<td>444.94</td>
<td>6.160</td>
<td>0.696</td>
<td>46.59</td>
<td>3.35</td>
<td>1.5809E-03</td>
<td>3.061642%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N\textperiodcentered m)</th>
<th>(\omega) (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m\textsuperscript{3}/s)\textsuperscript{2}</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>45.1</td>
<td>1202.27</td>
<td>2.535</td>
<td>0.286</td>
<td>125.90</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>3.168011%</td>
</tr>
<tr>
<td>45.2</td>
<td>976.06</td>
<td>3.374</td>
<td>0.381</td>
<td>102.21</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>3.423486%</td>
</tr>
<tr>
<td>45.3</td>
<td>836.11</td>
<td>4.076</td>
<td>0.461</td>
<td>87.56</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>3.542563%</td>
</tr>
<tr>
<td>45.4</td>
<td>757.17</td>
<td>5.091</td>
<td>0.575</td>
<td>79.29</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>4.007095%</td>
</tr>
<tr>
<td>45.5</td>
<td>647.03</td>
<td>5.792</td>
<td>0.654</td>
<td>67.76</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>3.895761%</td>
</tr>
<tr>
<td>45.6</td>
<td>501.51</td>
<td>6.413</td>
<td>0.725</td>
<td>52.52</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>3.343113%</td>
</tr>
<tr>
<td>45.7</td>
<td>401.82</td>
<td>6.444</td>
<td>0.728</td>
<td>42.08</td>
<td>3.6</td>
<td>1.6988E-03</td>
<td>2.691365%</td>
</tr>
</tbody>
</table>
### Appendix A. Air Engine Efficiency (Continued)

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N*m)</th>
<th>$\omega$ (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m^3/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.1</td>
<td>1209.87</td>
<td>2.791</td>
<td>0.315</td>
<td>126.70</td>
<td>4</td>
<td>1.8876E-03</td>
<td>3.159114%</td>
</tr>
<tr>
<td>50.2</td>
<td>1055.88</td>
<td>3.993</td>
<td>0.451</td>
<td>110.57</td>
<td>4</td>
<td>1.8876E-03</td>
<td>3.944222%</td>
</tr>
<tr>
<td>50.3</td>
<td>866.38</td>
<td>4.643</td>
<td>0.525</td>
<td>90.73</td>
<td>4</td>
<td>1.8876E-03</td>
<td>3.763271%</td>
</tr>
<tr>
<td>50.4</td>
<td>675.46</td>
<td>4.939</td>
<td>0.558</td>
<td>70.73</td>
<td>4</td>
<td>1.8876E-03</td>
<td>3.120747%</td>
</tr>
<tr>
<td>50.5</td>
<td>510.23</td>
<td>6.589</td>
<td>0.744</td>
<td>53.43</td>
<td>4</td>
<td>1.8876E-03</td>
<td>3.145194%</td>
</tr>
<tr>
<td>50.6</td>
<td>431.38</td>
<td>7.153</td>
<td>0.808</td>
<td>45.17</td>
<td>4</td>
<td>1.8876E-03</td>
<td>2.886736%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N*m)</th>
<th>$\omega$ (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m^3/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.1</td>
<td>1282.49</td>
<td>3.075</td>
<td>0.347</td>
<td>134.30</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.353906%</td>
</tr>
<tr>
<td>55.2</td>
<td>1203.17</td>
<td>3.115</td>
<td>0.352</td>
<td>126.00</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.186971%</td>
</tr>
<tr>
<td>55.3</td>
<td>1069.88</td>
<td>4.106</td>
<td>0.464</td>
<td>112.04</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.735952%</td>
</tr>
<tr>
<td>55.4</td>
<td>870.17</td>
<td>3.839</td>
<td>0.434</td>
<td>91.12</td>
<td>4</td>
<td>2.0764E-03</td>
<td>2.840786%</td>
</tr>
<tr>
<td>55.5</td>
<td>714.49</td>
<td>5.572</td>
<td>0.630</td>
<td>74.82</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.385658%</td>
</tr>
<tr>
<td>55.6</td>
<td>603.43</td>
<td>6.298</td>
<td>0.712</td>
<td>63.19</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.232146%</td>
</tr>
<tr>
<td>55.7</td>
<td>482.97</td>
<td>7.162</td>
<td>0.809</td>
<td>50.58</td>
<td>4</td>
<td>2.0764E-03</td>
<td>2.941951%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>RPM</th>
<th>Torque</th>
<th>Torque (N*m)</th>
<th>$\omega$ (rad/sec)</th>
<th>air flow mass (CFM)</th>
<th>air flow mass (m^3/s)²</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.1</td>
<td>1274.52</td>
<td>2.988</td>
<td>0.338</td>
<td>133.47</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.238937%</td>
</tr>
<tr>
<td>63.2</td>
<td>1231.84</td>
<td>3.279</td>
<td>0.371</td>
<td>129.00</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.435522%</td>
</tr>
<tr>
<td>63.3</td>
<td>1114.05</td>
<td>3.436</td>
<td>0.388</td>
<td>116.66</td>
<td>4</td>
<td>2.0764E-03</td>
<td>3.255829%</td>
</tr>
</tbody>
</table>
## Appendix A. Air Engine Efficiency (Continued)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>63.4</td>
<td>985.89</td>
<td>5.231</td>
<td>0.591</td>
<td>103.24</td>
<td>4.4</td>
<td>2.0764E-03</td>
</tr>
<tr>
<td>63.5</td>
<td>780.75</td>
<td>5.547</td>
<td>0.627</td>
<td>81.76</td>
<td>4.4</td>
<td>2.0764E-03</td>
</tr>
<tr>
<td>63.6</td>
<td>580.18</td>
<td>5.915</td>
<td>0.668</td>
<td>60.76</td>
<td>4.4</td>
<td>2.0764E-03</td>
</tr>
<tr>
<td>63.7</td>
<td>510.05</td>
<td>6.907</td>
<td>0.780</td>
<td>53.41</td>
<td>4.4</td>
<td>2.0764E-03</td>
</tr>
<tr>
<td>63.8</td>
<td>418.60</td>
<td>7.321</td>
<td>0.827</td>
<td>43.84</td>
<td>4.4</td>
<td>2.0764E-03</td>
</tr>
</tbody>
</table>