



*About
the
Author*

Janet H. Caldwell *received her doctorate in Mathematics Education from the University of Pennsylvania in 1977, after having completed a dual major in mathematics and French at Rice University.*

She has been active in mathematics education at all levels, previously founding and directing McSiip (South Jersey Regional Mathematics, Computer, and Science Instructional Improvement Program) at Glassboro State College and currently serving as President of the Association of Mathematics Teachers of New Jersey.

Changing Views of Mathematics



Janet Caldwell

At the beginning of each course that I teach, I ask students to answer a few questions on an index card. In every course, one of those questions is, “What is mathematics?” I generally get about the same responses, no matter what the course title:

Math is numbers. Math is numbers and shapes. Math is figuring out answers. Math is adding, subtracting, multiplying, and dividing. Math is memorizing rules and procedures.

I’ve become intrigued by these fairly limited views of mathematics, and I’ve looked for published work on students’ views of mathematics. Here are a few of the comments I’ve found. From the *Arithmetic Teacher*, January 1988, in “Problem Solving and Mathematical Beliefs” by Martha L. Frank:

1. Mathematics is computation.
2. Mathematical problems should be quickly solved in a few steps.

3. The goal of doing mathematics is to obtain the right “answer”; the teacher always has it!

4. Mathematics is a set of facts, rules, and procedures; it is a “package” to be passively received.

5. Mathematics teachers are supposed to spend time explaining or “covering” material from the text, then verify that the students have received the knowledge by checking the students’ answers. (32–34)

From the *Mathematics Teacher*, October 1989, in “Beliefs and Their Influence on Mathematical Performance,” by Joe Garofolo:

1. Almost all mathematics problems can be solved by the direct application of the facts, rules, formulas, and procedures shown by the teacher or given by the text.

2. Mathematical thinking consists of being able to learn, remember, and apply facts, rules, formulas, and procedures.

3. Mathematics textbook exercises can be solved only by the methods presented in the book; moreover, such exercises must be solved by the methods presented in the section of the text in which they appear.

4. Only the mathematics to be tested is important and worth knowing.

5. Formulas are important, but an understanding of the derivations of such formulas is not useful or needed for anything.

6. Mathematics is created only by very prodigious and creative people; other people just try to learn what is handed down. (502–505)

Students seem to develop the view that mathematics is some mysterious domain that is accessible only to the select few. We encourage this view by using mathematics as a filtering mechanism—if you don’t pass calculus, you can’t graduate as a business major. Students believe that teachers

give rules for solving problems and exercises, which must be memorized and used. As early as second grade, they believe that these rules generally do not make any sense. They learn that if they do what they are told to do, they will get the right answers (at least most of the time), and then everyone will be happy.

No wonder my students seem to have so much difficulty in my classes! I ask them to actually *think* about what they are learning! I ask them to communicate about mathematics and reason about mathematics and connect what they are learning now to what they already know. I ask precalculus students to look at a number of graphs and find the patterns in their equations. I ask geometry students to use a computer program to collect data about the results of connecting the midpoints of a quadrilateral, make conjectures based on the data, and prove or disprove their conjectures.

Somehow, students' views of mathematics are very different from my own. I see mathematics as a much more active, analytical discipline. I have always loved mathematics because I didn't have to memorize things; new ideas just followed logically from concepts I already knew and understood. I love seeing the commonalities between seemingly disparate areas—how the matrix algebra I learned in high school is really related to vectors and the geometry of multi-dimensional spaces, how transformations can help students understand the graphs of functions, how the squares of numbers are just the areas of squares with sides of that length. I love to look for the patterns. I like exploring, wading into a problem and seeing “what if,” trying out some hypotheses, looking for some generalizations, and trying to prove my results logically. The best definition I've ever seen for mathematics is one in *Reshaping School Mathematics* by the Mathematical Sciences Education Board: mathematics is “the language and science of patterns.”

Changes are happening right now, however, in mathematics teaching and learning in the elementary and secondary grades as well as at the college and university levels—changes that

will help students view mathematics more positively, changes that will develop mathematical power and an I-can-figure-it-out attitude. The National Council of Teachers of Mathematics has issued a document called *Curriculum and Evaluation Standards for School Mathematics* that advocates a common core in mathematics for all students through the high school level, including algebra, geometry, data analysis, discrete mathematics, and optimization. These recommendations represent a major shift in emphasis from current mathematics programs, from a narrow computationally-oriented curriculum to a broader curriculum that includes experiences with several branches of mathematics. Instead of acquiring disparate pieces of knowledge on a rote basis, students must develop a connected conceptual framework, using knowledge as a tool for solving problems. Students must be actively involved in constructing mathematical ideas through exploring, investigating, discussing, and conjecturing. Problem solving must indeed be the central focus of instruction, with calculators used to lessen the computational load.

Perhaps the most relevant demonstration of these changes for us as state college faculty in New Jersey is the new High School Proficiency Test, required of all students for graduation from high school, beginning with the class of 1995. This test now requires students to use higher-order thinking skills, with questions from such areas as numerical operations, measurement and geometry, data analysis, patterns and relationships, and fundamentals of algebra. It is likely that this test will, by 1995, allow the use of calculators on at least part of the test, as will the College Board's Scholastic Aptitude Test.

College mathematics is changing, too. No longer is Intermediate Algebra the single required mathematics course for graduation. In fact, this course is no longer appropriate for the vast majority of our students. Much more appropriate is a course such as Glassboro's "Contemporary Mathematics" that shows students new developments in mathematics, that

highlights recurring themes in mathematics, and that broadens their understanding of what mathematics is. Precalculus and calculus curricula are being modified to meet the changing needs of our students and our society. Graphing calculators are being used in these classes to help students better understand the links between geometry and algebra. While calculators have long been used in all Glassboro mathematics classes (except developmental education), computers are also being used more frequently to help students explore mathematical situations. Mathematics is a living, breathing, changing discipline, currently experiencing explosive growth; we as mathematics faculty are trying to share these new ideas with our students.

Our society can not afford further generations of mathematically and scientifically illiterate citizens. The new automated workplace requires radically different skills of our college graduates. Business and industry leaders need employees who understand and can use statistics, communication skills, and problem solving. They need employees who can work in cooperative groups, can make decisions about production problems based on quantitative information, and can acquire new skills and behaviors on the job. What is needed is not disconnected rules, theorems, and techniques but rather the ability to think and reason, applying the power of mathematics automatically to problems that need to be solved. We must prepare our students to perceive patterns and solve unconventional problems.

Even faculty who do not teach mathematics convey their beliefs about mathematics to their students. If faculty are negative, so will the students be. If faculty view math as a requirement to be put off until the end of the college career since "it's not really very useful," students will echo faculty opinion. On the other hand, if faculty encourage students to take interesting, challenging mathematics classes, students will do so. If faculty show how mathematics is used in their own discipline and explain why students need to understand the mathematical concepts, it will motivate them to learn.

I wish that every student could view mathematics as did this little girl:

What I Learned in Math This Year: I learned that math is not just numbers. It is life. The reason I say that is because math can be made from anything, art, music, clothes, ice cream, even the ceiling. I could go on forever. Math is also thinking out things that confuse you. We need math. Without decimals, everybody would go bankrupt! I learned that it doesn't matter how old you are or what grade you are in, you can learn anything if you really put your mind to it. If math is taught right, it is fun and easy to remember. (From 1989 *NCTM Yearbook*, p. 174.)

